
حمل الآن
مجانا وحصريا
المراجعة رقم (1)
اختبار شهر مارس
ذاكرولى
RaNia SaYed
WWW.ZAKROLYUR.COM
حمل الان حصريا

15 Marks

Model (1)

5

1 Choose the correct answer:

a The S.S. of the equation: $x^2 = x$ in $\mathbb{R}$ is ..... ({0} , ∅ , {1} , {0,1})

b $0.002 \times 0.05 =$ ..... (10^{-5} , 10^{-4} , 10^{4} , 10^{5})

c If $2^x + 2^x + 2^x = 48$, then $x =$ ..... (2 , 4 , 6 , 12)

d If $(25)^2 - (15)^2 = 10x$, then $x =$ ..... (40 , 30 , 20 , 10)

e If $3^{x-1} = \sqrt[3]{\frac{1}{27}}$, then $x =$ ..... (0 , 1 , -1 , -2)

10

2 Answer each of the following:

a If $3^x = 27$, $4^{x+y} = 1$, calculate the value of x and y.

.....

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b Find the value of n where $n \in \mathbb{Z}$.

$$\frac{3^n \times 8^n}{(12)^{n+1}} = \frac{1}{3}$$

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.....

c Find the dimensions of a rectangle whose length is 4 cm more than its width and its area is 21 cm^2.

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d Use the factorization to get the value of $(77)^2 - (23)^2$.

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e Factorize the expression: $4\,a^4 + b^4$

.....

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15 Marks

Model (2)

5

1 Choose the correct answer:

a If Ali's age now is x years, then his age after 3 years will be ..... years.

$(3x\ ,\ x-3\ ,\ x\div 3\ ,\ x^3)$

b If $6^x = 11$, then $6^{x+1} =$ ..... . $(12\ ,\ 22\ ,\ 66\ ,\ 72)$

c The S.S. of the equation: $x^2 - 4 = 0$ in $\mathbb{R}$ is ..... . $(\{4\}\ ,\ \{4,-4\}\ ,\ \{2\}\ ,\ \{2,-2\})$

d The numerical value of the expression: $\frac{2^{2n+1} \times 5^{2n+1}}{10^{2n}}$ is ..... $(7\ ,\ 10\ ,\ 100\ ,\ \frac{1}{10})$

e $4^3 + 4^3 + 4^3 + 4^3 =$ ..... . $(4^3\ ,\ 4^4\ ,\ 4^{12}\ ,\ 4^8)$

10

2 Answer each of the following:

a Find in $\mathbb{R}$ the S.S. of $(2x-1)^2 + (x-1)^2 = 10$

.....

.....

b Find in $\mathbb{R}$ the S.S. of the equation: $25 \times 3^{x-1} = 9 \times 5^{x-1}$

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c If $x = \frac{\sqrt{3}}{2}$, $y = \frac{1}{\sqrt{3}}$ and $Z = \frac{\sqrt{2}}{2}$,

find the value of $x^2 + (xz)^2 \times y^2$

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d What is the real number if it is added to its square, the result will be 12?

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.....

e Use the factorization to get the value of 31×29.

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.....

15 Marks

Model (3)

1 Choose the correct answer:

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a If $2^{x-1} \times 3^{1-x} = \frac{9}{4}$, then $x =$ ..... . (-3 , –1 , 1 , 3)

b If $x^2 - y^2 = 16$, $y - x = 2$, then $x + y =$ ..... . (4 , 8 , –8 , 2)

c If $(\frac{2}{3})^3 = (\frac{3}{2})^x$, then $x =$ ..... . (3 , –3 , 5 , –5)

d The S.S. of the equation: $x(x - 3) = 5x$ in $\mathbb{R}$ is ..... . ({3} ,{0, 3, 5}, {3, 5}, {0, 8})

e If $a + b = 5$, $a - b = 4$, then $b^2 - a^2 =$ ..... . (–20 , –1 , 9 , 20)

2 Answer each of the following:

10

a Hatem is 4 years older than Hanan now, and the sum of the squares of their ages now is 26. Find their ages.

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b Simplify $\frac{4^{x+1} \times 9^{2-x}}{6^{2x}}$, then find the value of the result when $x = 1$

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c Find in $\mathbb{R}$ the S.S. of the following equation:

$y^2 - \frac{7y}{3} = -\frac{4}{3}$

.....

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.....

d Factorize the following perfectly: $x^5 - x^3 - x^2 + 1$

.....

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.....

e Simplify the expression: $4^{x-1} \times 2^{3x+2} \times (\frac{1}{2})^{3x}$, then find the value of the result if $2^x = 5$.

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15 Marks

Model (1)

5

1 Choose the correct answer:

a The S.S. of the equation: $x^2 = x$ in $\mathbb{R}$ is ..... . ({0} , ∅ , {1} , **{0,1}**)

b $0.002 \times 0.05 =$ ..... . (10^{-5} , **10^{-4}** , 10^{4} , 10^{5})

c If $2^x + 2^x + 2^x = 48$, then $x =$ ..... . (2 , **4** , 6 , 12)

d If $(25)^2 - (15)^2 = 10x$, then $x =$ ..... . (**40** , 30 , 20 , 10)

e If $3^{x-1} = \sqrt[3]{\frac{1}{27}}$, then $x =$ ..... . (**0** , 1 , -1 , -2)

10

2 Answer each of the following:

a If $3^x = 27$, $4^{x+y} = 1$, calculate the value of x and y.

$\because 3^x = 27$

$\because 3^x = 3^3$ $\quad\quad$ $\therefore x = 3$

$\because 4^{x+y} = 1$

$\therefore 4^{3+y} = 4^0$ $\quad\quad$ $\therefore y = -3$

b Find the value of n where $n \in \mathbb{Z}$

$$\frac{3^n \times 8^n}{(12)^{n+1}} = \frac{1}{3}$$

$$\because \frac{3^n \times 2^{3n}}{3^{n+1} \times 4^{n+1}} = \frac{1}{3}$$

$$\because \frac{3^n \times 2^{3n}}{3^{n+1} \times 2^{2n+2}} = \frac{1}{3}$$

$\therefore 3^{-1} \times 2^{n-2} = 3^{-1}$

$\therefore 2^{n-2} = 1$

$\therefore 2^{n-2} = 2^0$ $\quad\quad$ $\therefore n = 2$

c Find the dimensions of a rectangle whose length is 4 cm more than its width and its area is 21 cm^2.

Let the width of the rectangle be x cm.

$\therefore$ the length of the rectangle is $(x+4)$ cm

$\therefore x\,(x+4) = 21$ $\qquad\qquad$ $\therefore\ x^2 + 4x - 21 = 0$

$\therefore (x+7)(x-3) = 0$

$\therefore\ x + 7 = 0$, then $x = -7$ (refused)

or $x - 3 = 0$, then $x = 3$

$\therefore$ The width = 3 cm and the length = 7 cm

d Use the factorization to get the value of $(77)^2 - (23)^2$.

$(77)^2 - (23)^2 = (77 + 23)(77 - 23)$

$= 100 \times 54 = 5{,}400$

e Factorize the expression $4a^4 + b^4$

$\because 2 \times \sqrt{4a^4} \times \sqrt{b^4} = 4a^2b^2$

$\therefore 4a^4 + b^4 = 4a^4 + b^4 + 4a^2b^2 - 4a^2b^2$

$= (4a^4 + 4a^2b^2 + b^4) - 4a^2b^2$

$= (2a^2 + b^2)^2 - (2ab)^2$

$= (2a^2 + b^2 - 2ab)(2a^2 + b^2 + 2ab)$

15 Marks

Model (2)

5

1 Choose the correct answer:

a If Ali's age now is x years, then his age after 3 years will be ..... years.

$(3x , x-3 , x+3 , x^3)$

b If $6^x = 11$, then $6^{x+1} =$ ..... . (12 , 22 , **66** , 72)

c The S.S of the equation $x^2 - 4 = 0$ in $\mathbb{R}$ is ..... . ({4} , {4,-4} , {2} , **{2,-2}**)

d The numerical value of the expression $\frac{2^{2n+1} \times 5^{2n+1}}{10^{2n}}$ is ..... (7 , **10** , 100 , $\frac{1}{10}$)

e $4^3 + 4^3 + 4^3 + 4^3 =$ ..... . (4^3 , 4^4 , 4^{12} , 4^8)

10

2 Answer each of the following:

a Find in $\mathbb{R}$ the S.S of $(2x-1)^2 + (x-1)^2 = 10$

$\because (2x-1)^2 + (x-1)^2 = 10$

$\therefore 4x^2 - 4x + 1 + x^2 - 2x + 1 = 10$

$\therefore 5x^2 - 6x + 2 = 10$

$\therefore 5x^2 - 6x - 8 = 0$

$\therefore (5x+4)(x-2) = 0$

$\therefore x = \frac{-4}{5}$ and $x = 2$

Then the S.S $= \{\frac{-4}{5} , 2\}$

b Find in $\mathbb{R}$ the S.S of the equation: $25 \times 3^{x-1} = 9 \times 5^{x-1}$

$5^2 \times 3^{x-1} = 3^2 \times 5^{x-1}$

$x - 1 = 2$ $\quad \therefore x = 3$ $\quad \therefore$ the S.S. $= \{3\}$

c If $x = \frac{\sqrt{3}}{2}$, $y = \frac{1}{\sqrt{3}}$ and $Z = \frac{\sqrt{2}}{2}$,

find the value of $x^2 + (xz)^2 \times y^2$

$$\left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{\sqrt{3}}{2} \times \frac{\sqrt{2}}{2}\right)^2 \left(\frac{1}{\sqrt{3}}\right)^2$$

$$= \frac{3}{4} + \frac{3}{8} \times \frac{1}{3} = \frac{3}{4} + \frac{1}{8} = \frac{7}{8}$$

d What is the real number if it is added to its square the result will be 12 ?

Let the number be x

$x + x^2 = 12$

$x^2 + x - 12 = 0$

$(x + 4)(x - 3) = 0$

then $x = -4$ or $x = 3$

e Use factorization to get the value of 31×29

$31 \times 29 = (30 + 1)(30 - 1)$

$= 30^2 - 1^2 = 900 - 1 = 899$

15 Marks

Model (3)

5

1 Choose the correct answer:

a If $2^{x-1} \times 3^{1-x} = \frac{9}{4}$, then $x =$ ..... . (-3 , **–1** , 1 , 3)

b If $x^2 - y^2 = 16$, $y - x = 2$, then $x + y =$ ..... . (4 , 8 , **–8** , 2)

c If $(\frac{2}{3})^3 = (\frac{3}{2})^x$, then $x =$ ..... . (3 , **–3** , 5 , –5)

d The S.S of the equation $x(x-3) = 5x$ in $\mathbb{R}$ is ..... . ({3} ,{0,3,5} ,{3,5} ,**{0,8}**)

e If $a + b = 5$, $a - b = 4$, then $b^2 - a^2 =$ ..... . (**–20** , –1 , 9 , 20)

10

2 Answer each of the following:

a Hatem is 4 years older than Hanan now , and the sum of squares of their ages now is 26. Find their ages.

Let the age of Hatem now be x years.

$\therefore$ The age of Hanan now = $(x - 4)$ years

$\therefore x^2 + (x-4)^2 = 26$

$\therefore x^2 + x^2 - 8x + 16 - 26 = 0$

$\therefore 2x^2 - 8x - 10 = 0$

$\therefore x^2 - 4x - 5 = 0$

$\therefore (x-5)(x+1) = 0$

$\therefore$ then $x = -1$ (refused), $x = 5$

$\therefore$ the age of Hatem now is 5 years.

$\therefore$ the age of Hanan now is one year.

b Simplify $\frac{4^{x+1} \times 9^{2-x}}{6^{2x}}$, then find the value of the result when $x = 1$

$$\frac{4^{x+1} \times 9^{2-x}}{6^{2x}}$$

$$= \frac{2^{2x+2} \times 3^{4-2x}}{2^{2x} \times 3^{2x}}$$

$$= 2^2 \times 3^{4-4x} = 4 \times 3^{4-4x}$$

The value when $x = 1$ is $2^2 \times 3^{4-4(1)} = 4 \times 3^0 = 4$

c Find in $\mathbb{R}$ the S.S. of the following equation:

$$y^2 - \frac{7y}{3} = -\frac{4}{3}$$

$3y^2 - 7y = -4$

$3y^2 - 7y + 4 = 0$

$(3y - 4)(y - 1) = 0$

Then $y = \frac{4}{3}$ Or $y = 1$

S.S. $= \{\frac{4}{3}, 1\}$

d Factorize the following perfectly: $x^5 - x^3 - x^2 + 1$

$x^3(x^2 - 1) - (x^2 - 1) = (x^2 - 1)(x^3 - 1)$

$= (x - 1)(x + 1)(x - 1)(x^2 + x + 1)$

$= (x - 1)^2(x + 1)(x^2 + x + 1)$

e Simplify the expression: $4^{x-1} \times 2^{3x+2} \times (\frac{1}{2})^{3x}$, then find the value of the result if $2^x = 5$.

$4^{x-1} \times 2^{3x+2} \times (\frac{1}{2})^{3x}$

$2^{2x-2} \times 2^{3x+2} \times (\frac{1}{2})^{3x}$

$2^{5x} \times 2^{-3x} = 2^{2x}$

The value when $2^x = 5$ is $2^{2x} = (2^x)^2 = (5)^2 = 25$

15 Marks

Model (1)

5

1 Choose the correct answer:

a The trapezium in which the length of one of its parallel bases is 15 cm, its area is 108 cm^2 and its height is 8 cm, then the length of the other base is ..... cm.

(15 , 4 , 12 , 27)

b The projection of a point on a given straight line is a ..... .

(point, line segment, ray, straight line)

c All ..... are similar. (triangles, squares, rhombuses, rectangles)

d The area of a square whose side length 5 cm ..... the area of a square whose diagonal length 7 cm. (> , < , = , ≡)

e If $\Delta ABC \sim \Delta DEF$ and $AB = \frac{1}{5} DE$, then the perimeter of ΔABC = ... perimeter of ΔDEF.

(5 , 1 , $\frac{1}{5}$, $\frac{2}{5}$)

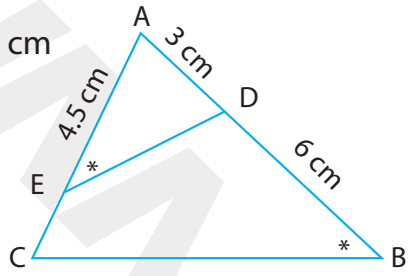
10

2 Answer each of the following:

a In the opposite figure:

$m(\angle AED) = m(\angle B)$, AD = 3 cm, AE = 4.5 cm and BD = 6 cm

- Prove that $\Delta ADE \sim \Delta ACB$
- Find the length of $\overline{EC}$



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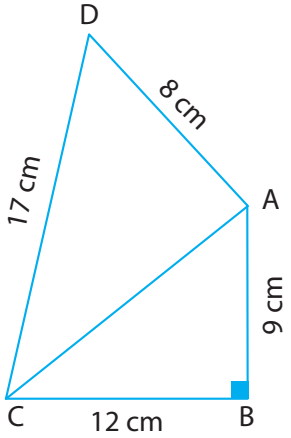
b In the opposite figure:

ABCD is a quadrilateral in which: $m(\angle B) = 90°$,
AB = 9 cm, BC = 12 cm, CD = 17 cm and DA = 8 cm
Prove that $m(\angle DAC) = 90°$



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c In the opposite figure:

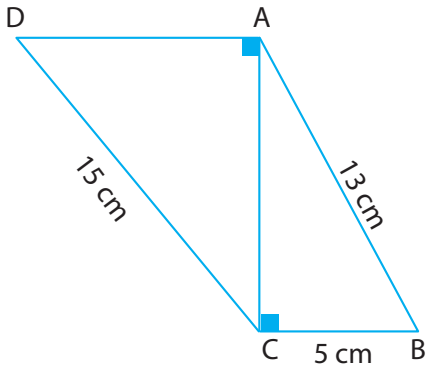
$\overline{AD}$ // $\overline{BC}$, AB = 13 cm, BC = 5 cm, CD = 15 cm

and m(∠ACB) = m(∠CAD) = 90°

Find:

– The length of the projection of $\overline{AB}$ on $\overleftrightarrow{AC}$

– The length of the projection of $\overline{CD}$ on $\overleftrightarrow{AD}$



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d If the ratio between the two lengths of the diagonals of a rhombus is 3:4 and the length of the smaller diagonal is 9cm. Find the area of the rhombus.

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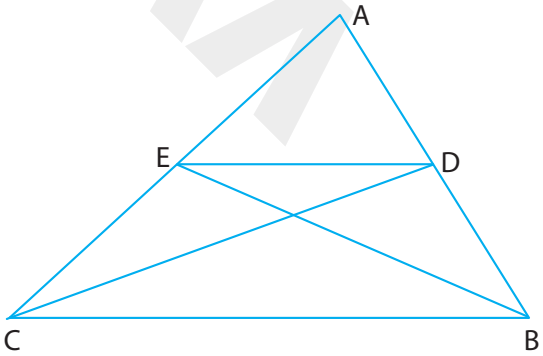
e ABC is a triangle in which D ∈ $\overline{AB}$ and E ∈ $\overline{AC}$,

such that the area of ΔABE = the area of ΔACD

Prove that: $\overline{DE}$ // $\overline{BC}$



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15 Marks

Model (2)

1 Choose the correct answer:

5

a The area of a square is 50 cm^2, then the length of its diagonal = ... cm.

(5 , 10 , 15 , 20)

b If the ratio between the lengths of two corresponding sides of two squares is 1 and the perimeter of one of them is 20 cm, then the area of the other square = ... cm^2.

(20 , 25 , 16 , 24)

c The projection of a line segment on the straight line not perpendicular to it is a ..... .

(ray , point , line segment , straight line)

d If the base length of a parallelogram is 7 cm and the corresponding height is 4 cm, then its area = ... cm^2. (11 , 14 , 22 , 28)

e If $\Delta XYZ \sim \Delta LMN$, then $\frac{\text{the perimeter of } \Delta XYZ}{\text{the perimeter of } \Delta LMN} =$ ..... .

($\frac{XY}{LM}$, $\frac{XZ}{YX}$, $\frac{NM}{ZY}$, $\left(\frac{XY}{LM}\right)^2$)

2 Answer each of the following:

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a A rhombus with diagonals lengths 12 cm and 16 cm. Find its side length, then its area.

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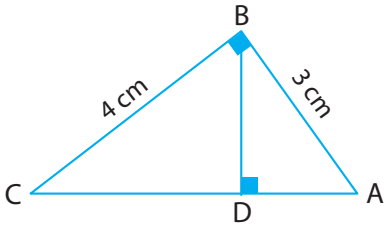
b **In the opposite figure:**

ABC is a right-angled triangle at B in which:

AB = 3 cm, BC = 4 cm, and $\overline{BD} \perp \overline{AC}$

- Prove that: Δ BAC ~ ΔDAB

- Find the length of $\overline{AD}$ and $\overline{DC}$



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c **In the opposite figure:**

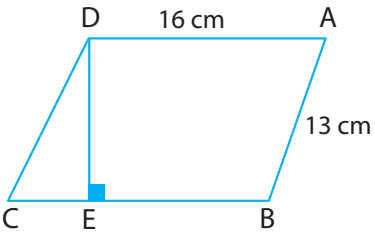
ABCD is a parallelogram in which:

AD = 16 cm and AB = 13 cm

If $\overline{DE} \perp \overline{BC}$ and the area of the parallelogram

ABCD = 192cm^2.

Find the length of the projection of $\overline{DC}$ on $\overleftrightarrow{BC}$



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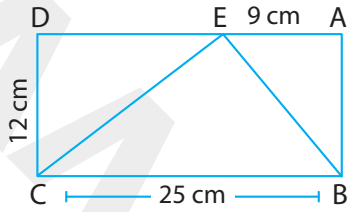
d **In the opposite figure :**

ABCD is a rectangle in which:

DC = 12 cm , AD = 25 cm and E ∈ $\overline{AD}$

Such that: AE = 9 cm

Prove that: $\overline{BE} \perp \overline{EC}$



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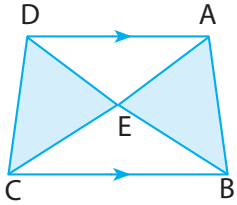
e In the opposite figure: ABCD is a quadrilateral,

$\overline{AD}$ // $\overline{BC}$, $\overline{AC} \cap \overline{BD} = \{E\}$

Prove that: the area of Δ ABE = the area of Δ DCE



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15 Marks

Model (3)

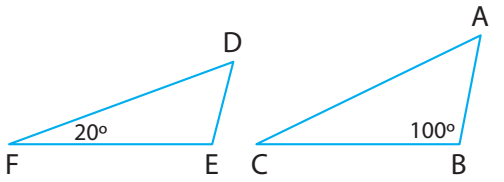
5

1 Choose the correct answer:

a In the opposite figure:

If Δ ABC ~ Δ DEF, then m(∠ A) = ... ° .

(20 , 60 , 80 , 100)



b The length of the projection of a line segment on a given straight line ..... of the line segment itself.

(> , ≥ ,≤ , =)

c If the base length of a triangle is 4 cm and the corresponding height is 3 cm. then its area = ... cm^2.

(6 , 12 , 24 , 30)

d If the perimeter of a rhombus = 48 cm and its area = 60 cm^2, then its height = ... cm.

(4 , 5 , 6 , 12)

e The projection of a ray on a straight line not perpendicular to it is ..... .

(point , line segment , ray , straight line)

10

2 Answer each of the following:

a The area of a trapezium is 180cm^2, and its height is 12 cm. Find the lengths of its parallel bases if the ratio between their lengths is 3 : 2

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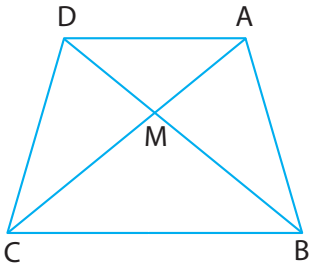
b In the opposite figure:

ABCD is a quadrilateral, its diagonals intersect at M and the area of ΔABM = The area of ΔDCM

Prove that: $\overline{AD}$ // $\overline{BC}$



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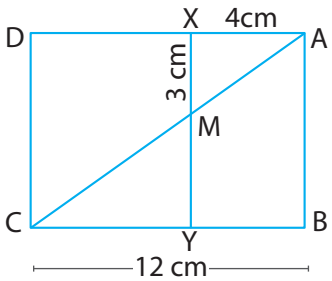
c **In the opposite figure:**

ABCD is a rectangle in which AD = 12 cm and

$X \in \overline{AD}$, where $AX = 4$ cm, $\overline{XY} \parallel \overline{AB}$ and

intersects $\overline{AC}$ at M and $\overline{BC}$ at Y , where MX = 3 cm

Prove that $\Delta AMX \sim \Delta CMY$



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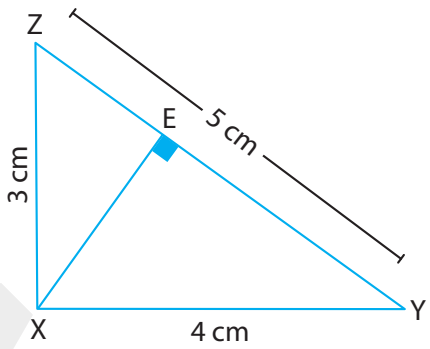
d **In the opposite figure:**

XYZ is a triangle in which $\overline{XE} \perp \overline{YZ}$,

$E \in \overline{ZY}$, YZ = 5 cm, XZ = 3 cm and

XY = 4 cm. Find the area of ΔXYZ ,

then find the length of $\overline{XE}$



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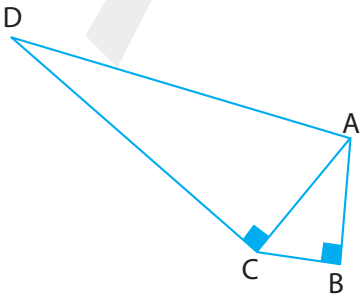
e **In the opposite figure:**

m (∠B) = m(∠ACD) = 90°



Complete:

a) The projection of $\overline{AD}$ on $\overleftrightarrow{CD}$ is ..... .

b) The projection of $\overline{AC}$ on $\overleftrightarrow{CD}$ is ..... .

c) The projection of $\overline{AC}$ on $\overleftrightarrow{AB}$ is ..... .

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15 Marks

Model (1)

5

1 Choose the correct answer:

a The trapezium in which the length of one of its parallel bases is 15 cm, its area is 108 cm^2 and its height is 8 cm, then the length of the other base is ..... cm.

(15 , 4 , **12** , 27)

b The projection of a point on a given straight line is a ..... .

(**point** , line segment , ray , straight line)

c All ..... are similar. (triangles , **squares** , rhombuses , rectangles)

d The area of a square whose side length 5 cm ..... the area of a square whose diagonal length 7 cm. (**>** , < , = , ≡)

e If $\Delta ABC \sim \Delta DEF$ and $AB = \frac{1}{5} DE$, then the perimeter of ΔABC = ... perimeter of ΔDEF.

(5 , 1 , $\frac{1}{5}$, $\frac{2}{5}$)

10

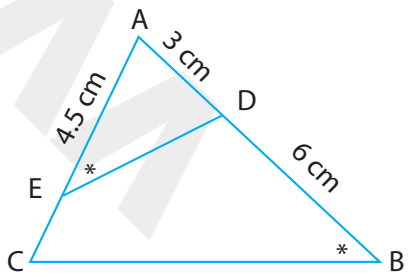
2 Answer each of the following:

a In the opposite figure:

$m(\angle AED) = m(\angle B)$, AD = 3 cm, AE = 4.5 cm and BD = 6 cm

- Prove that $\Delta ADE \sim \Delta ACB$

- Find the length of $\overline{EC}$



Answer Proof:

In ΔΔABC,AED

$\because m(\angle B) = m(\angle AED)$, $\angle A$ is a common angle

$\therefore m(\angle C) = m(\angle ADE)$

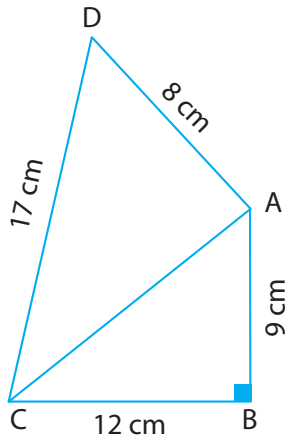
$\therefore \Delta ADE \sim \Delta ACB$ (1st req)

$\therefore \frac{AD}{AC} = \frac{AE}{AB}$ $\therefore \frac{3}{AC} = \frac{4.5}{9}$

$\therefore AC = \frac{3 \times 9}{4.5} = 6$ cm

$\therefore EC = 6 - 4.5 = 1.5$ cm (2nd req)

b In the opposite figure:



ABCD is a quadrilateral in which: m(∠B) = 90°,

AB = 9 cm, BC = 12 cm, CD = 17 cm and DA = 8 cm

Prove that m(∠DAC) = 90°

Answer Proof:

In ΔABC

∵ m(∠B) = 90°

∴ $(AC)^2 = (AB)^2 + (BC)^2 = 81 + 144 = 225$

∴ AC = 15 cm

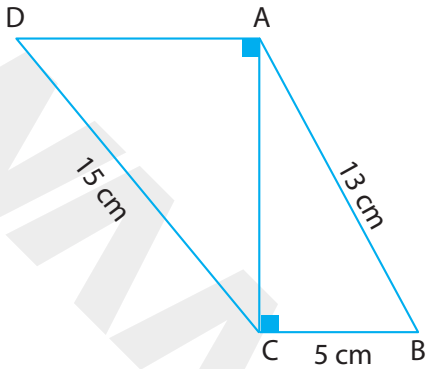
In ΔDAC

∵ $(AC)^2 = 225$, $(AD)^2 = 64$, $(DC)^2 = 289$

∴ $(DC)^2 = (AD)^2 + (AC)^2$

∴ m(∠DAC) = 90°

c In the opposite figure:



$\overline{AD}$ // $\overline{BC}$, AB = 13 cm, BC = 5 cm, CD = 15 cm

and m(∠ACB) = m(∠CAD) = 90°

Find :

– The length of the projection of $\overline{AB}$ on $\overleftrightarrow{AC}$

– The length of the projection of $\overline{CD}$ on $\overleftrightarrow{AD}$

Answer:

∵ $\overline{AC}$ is the projection of $\overline{AB}$ on $\overleftrightarrow{AC}$

In the right-angled triangle ACB at C

∵ $(AC)^2 = (AB)^2 - (BC)^2 = 169 - 25 = 144$

∴ AC = 12 cm (first req.)

$\overline{AD}$ is the projection of $\overline{CD}$ on $\overleftrightarrow{AD}$

In the right-angled triangle DAC at A

∵ $(AD)^2 = (CD)^2 - (AC)^2 = 225 - 144 = 81$

∴ AD = 9 cm (second req.)

d If the ratio between the two lengths of the diagonals of a rhombus is 3:4 and the length of the smaller diagonal is 9cm. find the area of the rhombus

Answer:

Let the length of the smaller diagonal be $3x$ cm

$\therefore$ The length of the greater diagonal = $4x$ cm

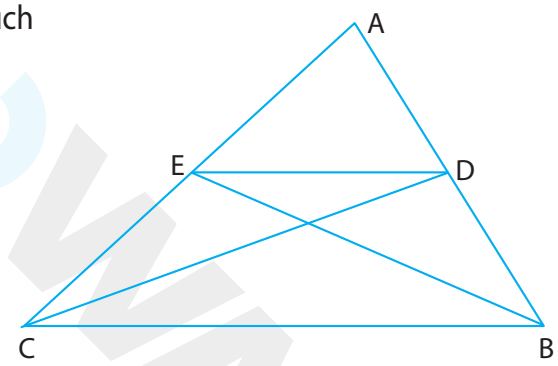
$\therefore 3x = 9$ $\therefore x = 3$

$\therefore$ The length of the greater diagonal = $4 \times 3 = 12$ cm

$\therefore$ The area of the rhombus = $\frac{1}{2} \times 9 \times 12 = 54$ cm^2

e ABC is a triangle in which D $\in \overline{AB}$ and E $\in \overline{AC}$, such that the area of ΔABE = the area of ΔACD

Prove that: $\overline{DE} // \overline{BC}$



Answer Proof:

$\because$ The area of ΔABE = The area of ΔACD

(By subtracting the area of ΔADE from both sides)

$\therefore$ The area of ΔDEB = The area of ΔDEC

and they have a common base $\overline{DE}$ and on one side of it

$\therefore \overline{DE} // \overline{BC}$

15 Marks

Model (2)

1 Choose the correct answer: (5)

a The area of a square is 50 cm^2, then the length of its diagonal = ... cm.

(5 , **10** , 15 , 20)

b If the ratio between the lengths of two corresponding sides of two squares is 1 and the perimeter of one of them is 20 cm, then the area of the other square = ... cm^2.

(20 , **25** , 16 , 24)

c The projection of a line segment on the straight line not perpendicular to it is a ..... .

(ray , point , **line segment** , straight line)

d If the base length of a parallelogram is 7 cm and the corresponding height is 4 cm, then its area = ... cm^2.

(11 , 14 , 22 , **28**)

e If $\Delta XYZ \sim \Delta LMN$, then $\frac{\text{the perimeter of } \Delta XYZ}{\text{the perimeter of } \Delta LMN}$ = ...

($\frac{XY}{LM}$, $\frac{XZ}{YX}$, $\frac{NM}{ZY}$, $\left(\frac{XY}{LM}\right)^2$)

2 Answer each of the following: (10)

a A rhombus with diagonals lengths are 12 cm and 16 cm. Find its side length, then its area.

Answer:

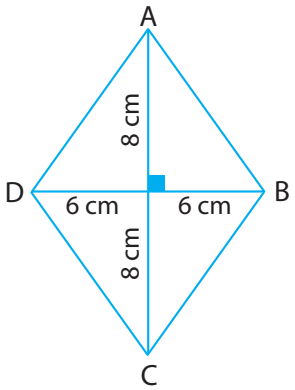
From the figure:

$(AB)^2 = (6)^2 + (8)^2 = 100$

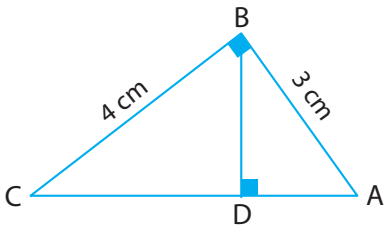
$\therefore$ AB = 10 cm

$\therefore$ The side length = 10 cm

$\therefore$ The area of the rhombus = $\frac{1}{2} \times 12 \times 16 = 96\ cm^2$



b In the opposite figure:



ABC is a right-angled triangle at B in which:

AB = 3 cm, BC = 4 cm and $\overline{BD} \perp \overline{AC}$

– **Prove that:** ΔBAC ~ ΔDAB

– **Find** the length of $\overline{AD}$ and $\overline{DC}$

Answer Proof:

In ΔΔBAC, DAB

$\because$ m(∠ABC) = m(∠ADB) = 90°, ∠A is a common angle

$\therefore$ m(∠C) = m(∠ABD)

$\therefore$ Δ BAC ~ ΔDAB (first req.)

$\because$ Δ ABC is right - angled at B

$\therefore (AC)^2 = (AB)^2 + (BC)^2 = 9 + 16 = 25$

$\therefore$ AC = 5 cm

$$\therefore \frac{AD}{AB} = \frac{AB}{AC} = \frac{BD}{CB}$$

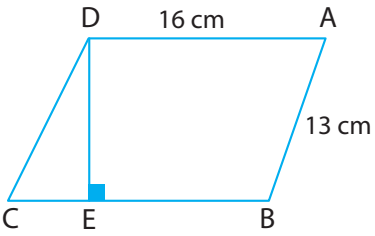
$$\therefore \frac{AD}{3} = \frac{3}{5} = \frac{BD}{4}$$

$$\therefore AD = \frac{3 \times 3}{5} = 1.8 \text{ cm}$$

$\because$ DC = AC – AD

$\therefore$ DC = 5 – 1.8 = 3.2 cm

c In the opposite figure:



ABCD is a parallelogram in which:

AD = 16 cm and AB = 13 cm

If $\overline{DE} \perp \overline{BC}$ and the area of the parallelogram

ABCD = 192cm^2.

Find the length of the projection of $\overline{DC}$ on $\overleftrightarrow{BC}$

Answer Proof:

$\overline{EC}$ is the projection of $\overline{DC}$ on $\overleftrightarrow{BC}$

$\because$ ABCD is a parallelogram $\therefore$ AB = DC

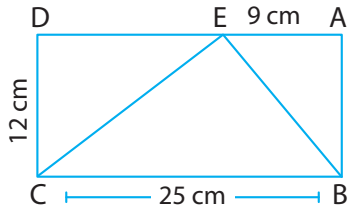
$\therefore$ DC = 13 cm , DE = $\frac{192}{16}$ =12 cm

In the right-angled triangle DEC at E

$(EC)^2 = (DC)^2 - (DE)^2 = 169 - 144 = 25$

$\therefore$ EC = 5 cm

d In the opposite figure:



ABCD is a rectangle in which:

DC = 12 cm , AD = 25 cm and E $\in \overline{AD}$

Such that: AE = 9 cm

Prove that: $\overline{BE} \perp \overline{EC}$

Answer Proof:

$\because$ ABCD is a rectangle

$\therefore$ ΔBAE is right-angled at A

$\therefore (EB)^2 = (AE)^2 + (AB)^2 = 81 + 144 = 225$

$\therefore$ EB = 15 cm

$\therefore$ ΔEDC is a right-angled at D

$\therefore$ ED = AD – AE = 25 – 9 = 16 cm

$\therefore (EC)^2 = (ED)^2 + (DC)^2 = 256 + 144 = 400$

$\therefore$ EC = 20 cm

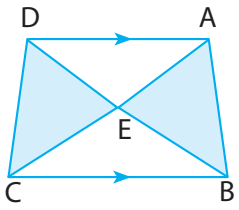
In ΔBEC

$\therefore (BC)^2 = 625 , (BE)^2 = 225 , (EC)^2 = 400$

$\therefore (BC)^2 = (BE)^2 + (EC)^2$, then m ($\angle$BEC) = 90°

$\therefore \overline{BE} \perp \overline{EC}$

e In the opposite figure : ABCD is a quadrilateral,



$\overline{AD}$ // $\overline{BC}$, $\overline{AC} \cap \overline{BD} = \{E\}$

Prove that : the area of Δ ABE = the area of Δ DCE

Answer Proof:

∵ Δ Δ ADB , ADC have a common base $\overline{AD}$, $\overline{AD}$ // $\overline{BC}$

∴ The area of triangle ADB = The area of triangle ADC

(By subtracting the area of triangle AED from both sides)

∴ The area of triangle ABE = The area of triangle DCE

15 Marks

Model (3)

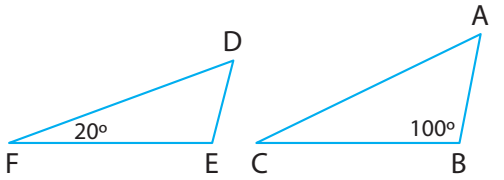
5

1 Choose the correct answer:

a In the opposite figure:

If ΔABC ~ ΔDEF, then m(∠A) = ... ° .

(20 , **60** , 80 , 100)



b The length of the projection of a line segment on a given straight line ..... of the line segment itself .

(> , ≥ , ≤ , =)

c If the base length of a triangle is 4 cm and the corresponding height is 3 cm . then its area = ... cm^2 .

(**6** , 12 , 24 , 30)

d If the perimeter of a rhombus 48 cm and its area = $60cm^2$, then its height = ... cm .

(4 , **5** , 6 , 12)

e The projection of a ray on a straight line not perpendicular to it a ..... .

(point , line segment , **ray** , straight line)

10

2 Answer each of the following:

a The area of a trapezium is $180cm^2$, and its height is 12 cm. Find the lengths of its parallel bases if the ratio between their lengths is 3:2

Answer:

Let the lengths of the two parallel bases be $3x$ cm and $2x$ cm

$\because$ The area $= \frac{1}{2}(3x + 2x) \times 12$

$\therefore 180 = \frac{1}{2}(3x + 2x) \times 12$

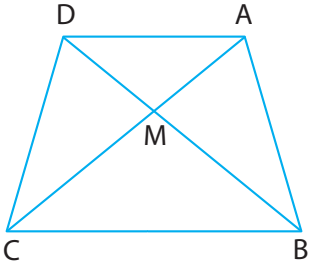
$\therefore 30x = 180 \qquad \therefore x = 6$ cm

$\therefore$ The lengths of the two bases are 18 cm and 12 cm

b In the opposite figure:

ABCD is a quadrilateral, its diagonals intersect at M and the area of ΔABM = The area of ΔDCM

Prove that: $\overline{AD} \parallel \overline{BC}$



Answer Proof

∵ The area of ΔABM= The area of ΔDMC

(By adding the area of ΔBMC to both sides)

∴ The area of ΔABC = The area of ΔDCB

(and they have the common base $\overline{BC}$ and on one side of it)

∴ $\overline{AD}$ // $\overline{BC}$

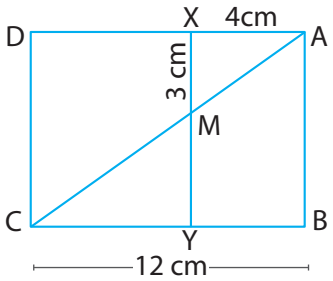
c In the opposite figure:

ABCD is a rectangle in which AD = 12 cm and

$X \in \overline{AD}$, where AX = 4 cm, $\overline{XY}$ // $\overline{AB}$ and

intersects $\overline{AC}$ at M and $\overline{BC}$ at Y , where MX = 3 cm

Prove that ΔAMX ~ ΔCMY



Answer Proof

∵ $\overline{XY}$ // $\overline{AB}$, $\overline{AX}$ // $\overline{BY}$

∴ ABYX is a parallelogram

∵ m(∠B) = 90° ∴ ABYX is a rectangle

∴ BY = AX = 4 cm

∵ BC = AD = 12 cm

∴ YC = 12 – 4 = 8 cm

∵ AXM is a right-angled triangle at X

∵ $(AM)^2 = (AX)^2 + (XM)^2 = 16 + 9 = 25$

AM = 5 cm

In ΔΔAMX , CMY

∵ m(∠AXM) = m(∠MYC) = 90°

∵ m(∠AMX) = m(∠CMY) (V.O.A)

∴ m(∠XAM) = m(∠MCY)

∴ ΔAMX ~ ΔCMY

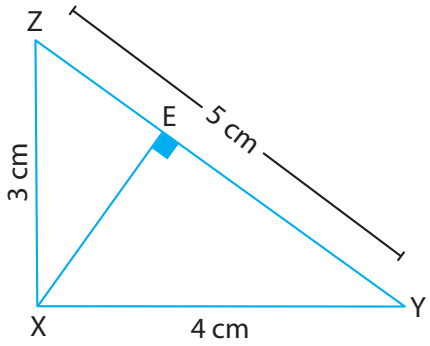
d In the opposite figure:

XYZ is a triangle in which $\overline{XE} \perp \overline{YZ}$,

$E \in \overline{ZY}$, YZ = 5 cm, XZ = 3 cm and

XY = 4 cm. Find the area of ΔXYZ ,

then find the length of $\overline{XE}$



Answer

In ΔXYZ

$\because (YZ)^2 = 25 , (XY)^2 = 16 , (ZX)^2 = 9$

$\therefore (YZ)^2 = (XY)^2 + (ZX)^2$

$\therefore m(\angle YXZ) = 90°$

$\therefore$ The area of ΔXYZ $= \frac{1}{2} \times 4 \times 3 = 6\ cm^2$ (first req)

$\because \overline{XE} \perp \overline{YZ}$

$\because XE = \frac{3 \times 4}{5} = 2.4$ cm (second req)

e In the opposite figure:

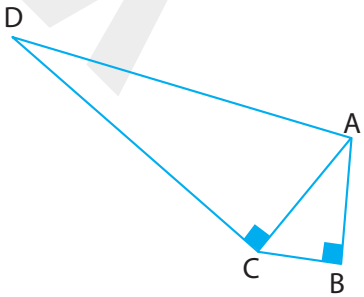
$m(\angle B) = m(\angle ACD) = 90°$

Complete:

The projection of $\overline{AD}$ on $\overleftrightarrow{CD}$ is ..... .

The projection of $\overline{AC}$ on $\overleftrightarrow{CD}$ is ..... .

The projection of $\overline{AC}$ on $\overleftrightarrow{AB}$ is ..... .



Answer

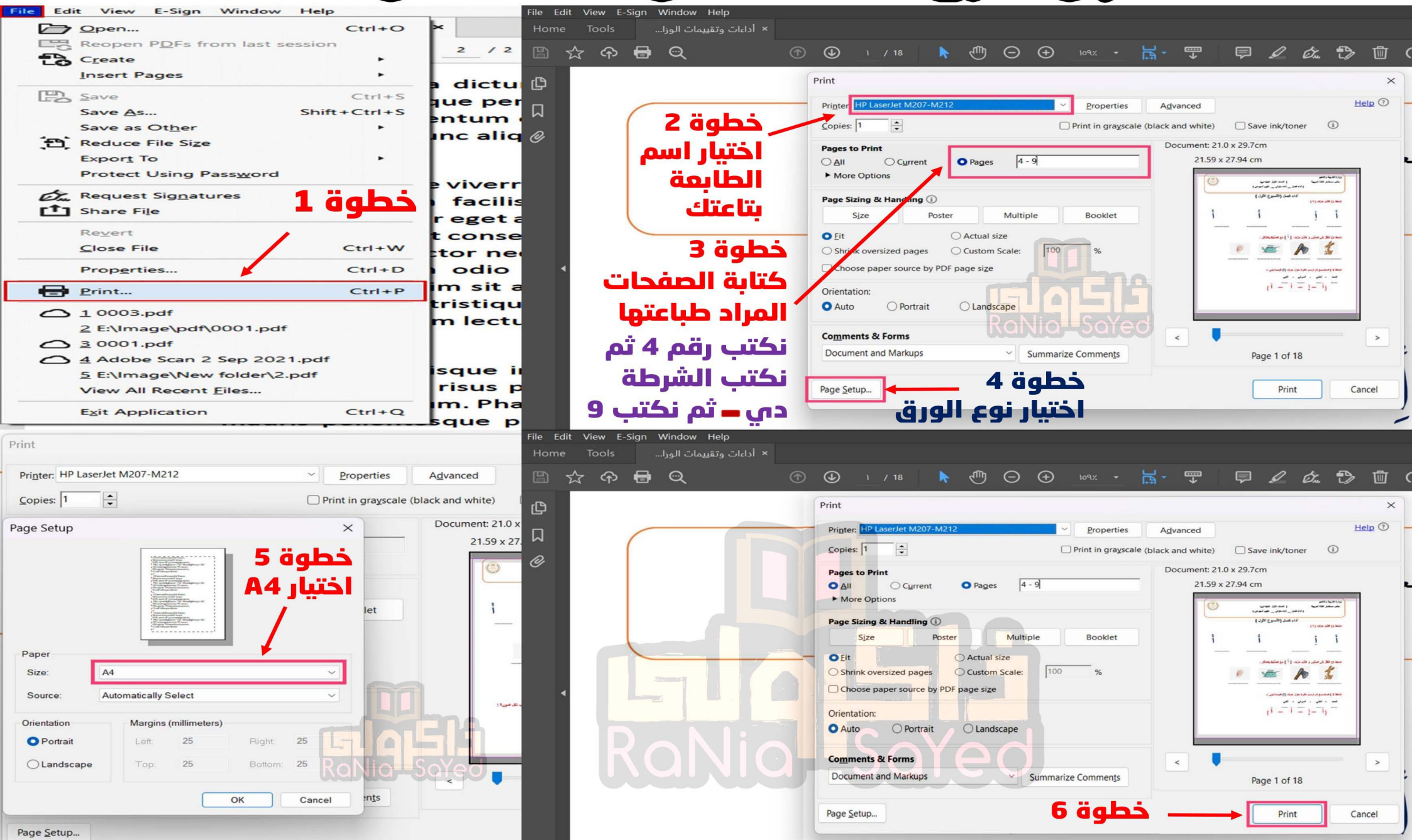
a) $\overline{CD}$

c) The point C

d) $\overline{AB}$

كيفية طباعة صفحات معينة من ملف معين

مثلا ازاى نطبع الصفحات من صفحة 4 الى صفحة 9



المراجعة رقم (2)

اختبار شهر مارس

April tests on Algebra and Statistics

Test 1

Total mark: 10

Answer the following questions :

1 Choose the correct answer from the given ones : (3 Marks)

1 The additive inverse of $\left(\sqrt{3}\right)^{-4}$ is .....

(a) $\frac{1}{9}$ (b) $-\frac{1}{9}$ (c) $\left(\sqrt{3}\right)^{4}$ (d) $\left(-\sqrt{3}\right)^{4}$

2 $5\ a^{0}$ = ... where $a \neq 0$

(a) 5 (b) 1 (c) a (d) 5 a

3 If $2^{x} = 7$, $2^{y} = 5$, then 2^{x-y} = ...

(a) 35 (b) $\frac{7}{5}$ (c) 2 (d) 12

2 Complete : (3 Marks)

1 If $3^{x+3} = 1$, then $2x$ = ...

2 $\left(\sqrt{7}\right)^{3} \times \left(\sqrt{7}\right)^{5} = 7^{.....}$

3 Four times the number 2^{8} is .....

3 Simplify : $\frac{4^{n} \times 6^{2n}}{3^{2n} \times 2^{4n}}$ (2 Marks)

4 If $3^{x} = 27$, $4^{x+y} = 1$ (2 Marks)

Find : The value of each of x , y

Test 2

Total mark: 10

Answer the following questions :

1 Choose the correct answer from the given ones : (3 Marks)

1 If $2^{x} = 11$, then $2^{x+1} =$ .....

(a) 22 (b) 12 (c) 112 (d) 212

2 $0.004 \times 0.00025 = 10^{.....}$

(a) 6 (b) 100 (c) 5 (d) – 6

3 $3^{x} + 3^{x} + 3^{x} = 1$, then $x =$ .....

(a) 3 (b) – 1 (c) – 3 (d) 1

---

2 Complete : (3 Marks)

1 $\left(\sqrt{3}+\sqrt{2}\right)^{10}\left(\sqrt{3}-\sqrt{2}\right)^{10} =$ .....

2 The multiplicative inverse of $\left|\left(\frac{2}{5}\right)^{-3}\right|$ is .....

3 If $7^{x-2} = 5^{x-2}$, then $x =$ .....

---

3 Find in $\mathbb{R}$ the S.S. of the equation : $(x-2)^5 = 32$ (2 Marks)

---

4 Prove that : $\frac{9^{x-1} \times 4^{x}}{6^{2x}} = 9$ (2 Marks)

April tests on Geometry

Test 1

Total mark: 10

Answer the following questions :

1 Choose the correct answer from the given ones : (3 Marks)

1 The two triangles are similar if the corresponding ..... are proportional.

(a) sides (b) angles (c) vertices (d) diagonals

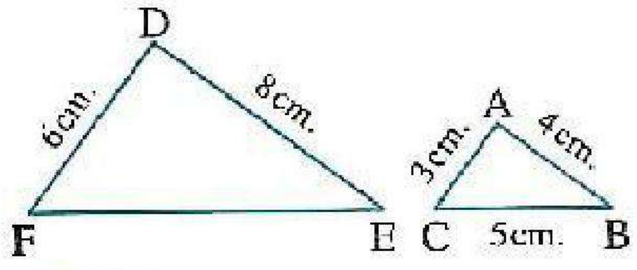
2 The length of the projection of a line segment on a given straight line ..... the length of the original line segment.

(a) ≥ (b) ≤ (c) > (d) =

3 **In the oppoiste figure :**

If Δ ABC ~ Δ DEF

, then the perimeter of Δ DEF =..... cm.



(a) 10 (b) 12 (c) 24 (d) 26

2 Complete : (3 Marks)

1 In a triangle , if the square of the length of a side is equal to the sum of the squares of the lengths of the other two sides , then .....

2 ABCD is a rectangle , then the projection of $\overline{AC}$ on $\overleftrightarrow{BC}$ is .....

3 If two polygons are similar and the ratio between the lengths of two corresponding sides is 5 : 8 , then the ratio between their perimeters is .....

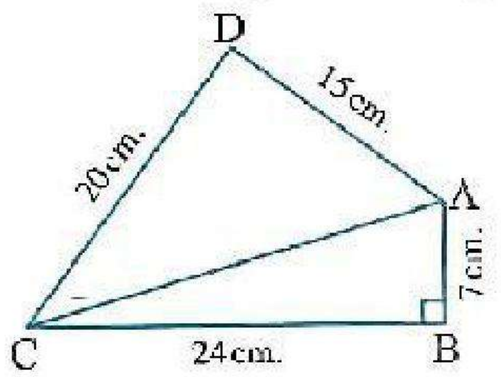
3 In the opposite figure : (2 Marks)

ABCD is a quadrilateral , m (∠ B) = 90°

, AB = 7 cm. , BC = 24 cm. , CD = 20 cm. , DA = 15 cm.

1 **Find :** The length of $\overline{AC}$

2 **Prove that :** m (∠ D) = 90°



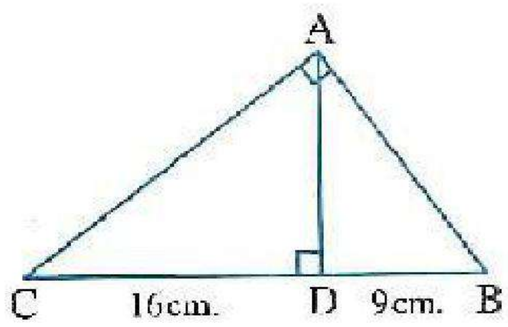
4 In the opposite figure : (2 Marks)

ABC is a right-angled triangle at A

, $\overline{AD} \perp \overline{BC}$

If BD = 9 cm. , CD = 16 cm.

Find the length of : $\overline{AB}$, $\overline{AC}$ and $\overline{AD}$



Test 2

Total mark: 10

Answer the following questions :

1 Choose the correct answer from the given ones : (3 Marks)

1 The projection of a ray on a straight line not perpendicular to it is .....

(a) a line segment. (b) a ray.

(c) a straight line. (d) a point.

2 All ..... are similar.

(a) rohmbuses (b) triangles (c) rectangles (d) squares

3 If the enlargement ratio of two simlilar polygons is ..... , then the two polygons are congruent.

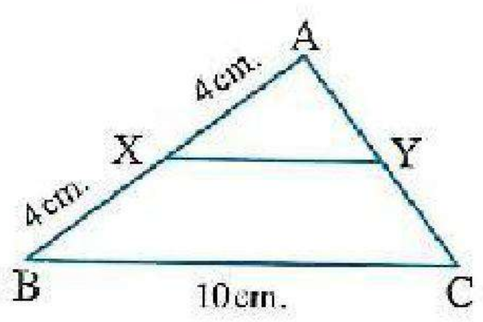
(a) 1 (b) 2 (c) 0.5 (d) otherwise

2 Complete : (3 Marks)

1 **In the opposite figure :**

If $\Delta ABC \sim \Delta AXY$, $AX = BX = 4$ cm.

, $BC = 10$ cm. , then $XY =$ ..... cm.



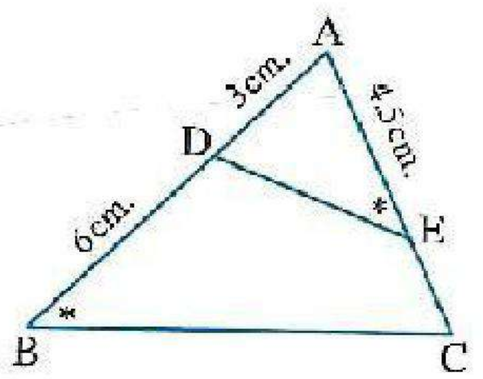
2 If $A \in \overline{BC}$, then the projection of A on $\overleftrightarrow{BC}$ is .....

3 In ΔXYZ , $(XY)^2 - (YZ)^2 = (XZ)^2$, then m ($\angle$ .....) = 90°

3 In the opposite figure : (2 Marks)

m ($\angle AED$) = m ($\angle B$) , $AD = 3$ cm.

, $AE = 4.5$ cm. , $BD = 6$ cm.



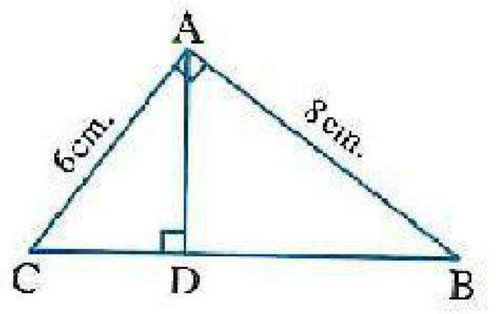
1 **Prove that :** $\Delta ABC \sim \Delta AED$

2 **Find :** The length of $\overline{EC}$

4 In the opposite figure : (2 Marks)

m ($\angle BAC$) = 90° , $\overline{AD} \perp \overline{BC}$

, $AB = 8$ cm. , $AC = 6$ cm.



Find : 1 The length of $\overline{BC}$

2 The length of the prejection of $\overline{AB}$ on $\overleftrightarrow{BC}$

المراجعة رقم (3)

اختبار شهر مارس

Lesson (7)
Factorizing by completing the square

The method of factorization by completing the square :

1. We add to the given expression twice the product of the two square roots of the two perfect square terms and subtract it again not to change the main expression.
2. Using the commutative and associative properties , we rewrite the expression after ordering its terms to get the form :
   a perfect square trinomial – a perfect square monomial
3. We factorize the resultant expression as a difference between two squares.
4. If it is possible , we should factorize the resultant expressions (resultant factors) in order that the factorization is perfect.

Factorize each of the following perfectly:

1. $x^4 + 4y^4$

.....

.....

.....

2. $a^4 + 2500\,b^4$

.....

.....

.....

3. $8x^4y^2 + 162z^4y^2$

.....

.....

.....

4. $x^4 + 9x^2 + 81$

.....

.....

5.

$m^4 - 11\ m^2n^2 + n^4$

.....

.....

.....

6. $4\ x^4 + 25\ y^4 - 29\ x^2\ y^2$

.....

.....

.....

Homework

1. $81\ x^4 + 4\ z^4$

.....

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2. $4\ x^4 + 625\ z^4$

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3. $9\ x^4 - 25\ x^2 + 16$

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4. $x^4 + x^2\ y^2 + 25\ y^4$

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5. $16\ x^4 - 28\ x^2\ y^2 + 9\ y^4$

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Lesson (8)
Solving quadratic equations in one variable algebraically

Complete each of the following :

1. If -5 is a root of the equation : $x^2 + 2x - 15 = 0$, then the other root is .....

2. If $x = 2$ is a root of the equation : $x^2 - 6x + k = 0$, then k = ... and the other root is .....

3. If one of the roots of the equation : $2x^2 + 8x = 0$ is a root of the equation : $x^2 + 5x + a = 0$, then a = ... or .....

Homework

1. If the number 9 is a solution of the equation : $x^2 + k = 0$, then k = ...

2. The solution set of the equation : $x^2 + 25 = 0$ in $\mathbb{R}$ is .....

3. The solution set of the equation $x^2 = 4x$ in $\mathbb{R}$ is .....

Choose the correct answer :

1. The S.S. of the equation : $3(x-2)(x+5) = 0$ in $\mathbb{R}$ is .....
   (a) $\{0, 2, -5\}$ (b) $\{3, 2, -5\}$ (c) $\{2, -5\}$ (d) $\{-2, 5\}$

2. The S.S. of the equation : $x^2 - 4 = 0$ in $\mathbb{R}$ is .....
   (a) $\{4\}$ (b) $\{4, -4\}$ (c) $\{2\}$ (d) $\{2, -2\}$

3. The S.S. of the equation : $x^2 + 25 = 0$ in $\mathbb{R}$ is .....
   (a) $\{5\}$ (b) $\{5, -5\}$ (c) $\{-5\}$ (d) $\varnothing$

4. The equation whose roots are 3 and 5 is .....
   (a) $5x^2 + 8x + 3 = 0$ (b) $2x^2 + 8x - 15 = 0$
   (c) $x^2 - 8x + 15 = 0$ (d) $3x^2 + 8x + 5 = 0$

5. The S.S. of the equation : $x(x-3) = 5x$ in $\mathbb{R}$ is .....
   (a) $\{3\}$ (b) $\{0, 3, 5\}$ (c) $\{3, 5\}$ (d) $\{0, 8\}$

6. The S.S. of the equation : $\frac{4}{x} = \frac{x}{9}$ in $\mathbb{R}$ is .....

(a) $\{4 , 9\}$ (b) $\{6 , -6\}$ (c) $\{6\}$ (d) $\{36\}$

7. If the number 4 is a solution of the equation : $x^2 + x - 20 = 0$, then the other solution is .....

(a) 20 (b) 5 (c) -5 (d) -4

Homework

1. The S.S. of the equation : $(x-4)^2 = 0$ in $\mathbb{R}$ is .....

(a) $\{4\}$ (b) $\{0 , 4\}$ (c) $\{0 , -4\}$ (d) $\{-4\}$

2. The solution set of the equation : $x(x-3) = 0$ in $\mathbb{R}$ is .....

(a) $\{3\}$ (b) $\{0 , 3\}$ (c) $\{0 , -3\}$ (d) $\{0\}$

3. If $3x^2 + c\,x - 6 = (3x-2)(x+3)$, then c = ...

(a) 7 (b) 12 (c) 13 (d) 5

4. The expression : $x^2 + 6x + a$ is a perfect square when a = ...

(a) 6 (b) 16 (c) 1 (d) 9

5. $x^3 + y^3 = (.....)(x^2 - xy + y^2)$

(a) $x^2 + y^2$ (b) $x^2 - y^2$ (c) $x + y$ (d) $x - y$

6. One of the factors of the expression : $x^2 - 3x - 18$ is .....

(a) $x - 3$ (b) $x - 6$ (c) $x - 9$ (d) $x - 18$

Find in R the solution set of each of the following equations:

1. $x^2 - 7x - 30 = 0$

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2. $2x^2 + 7x = 0$

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3. $(x+2)^2 = 25$

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4. $(x-3)(x+5)=20$

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5. $x-\frac{2}{x}=\frac{7}{2}$

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6. $x(x-1)=6$

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7. $3x^3=12x$

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8. $x^3-4x=0$

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9. $x^4-13x^2+36=0$

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10. If : $x^2+\frac{1}{x^2}=34$, then **find** : $x+\frac{1}{x}$

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11. If : $x + \frac{1}{x} = 2$, then find : $x^2 + \frac{1}{x^2}$

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Homework

1. $x^2 - 5x - 6 = 0$

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2. $x^2 - 6x = -9$

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3. $x - \frac{3}{x} = 2$

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4. $x^2 - 5x = 0$

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5. $4x^2 = 25$

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Lesson (9)
Applications on solving quadratic equations in one variable algebraically

Complete each of the following :

1. Twice the square of the number x is .....

2. If the age of Bassim now is x years , then his age 3 years ago was ..... years.

Choose the correct answer :

1. If the age of Ayman 5 years ago was x years , then the square of his age now = ...
(a) $x^2 + 5$ (b) $x^2 + 25$ (c) $(x + 5)^2$ (d) $(x - 5)^2$

2. If the age of Bassim now is x years , then his age 3 years ago was ..... years.
(a) $3x$ (b) $x + 3$ (c) $x - 3$ (d) x^3

3. If the age of Amgad now is x years , then his age after 7 years will be ..... years.
(a) $7x$ (b) $x - 7$ (c) $x + 7$ (d) x^7

4. If the age of Ayman 5 years ago was x years , then his age now is ..... years.
(a) $x - 5$ (b) $x + 5$ (c) $5x$ (d) $\frac{x}{5}$

5. If the age of Sally 2 years ago was x years , then her age after 3 years from now will be ..... years.
(a) $x + 2$ (b) $x + 3$ (c) $x + 5$ (d) $6x$

6. If the age of Magdy now is x years , then the square of his age after 2 years is .....
(a) $x^2 + 2$ (b) $x^2 + 4$ (c) $(x - 2)^2$ (d) $(x + 2)^2$

7. If the age of Samy now is x years , then twice his age 5 years ago is ..... years.
(a) $x - 5$ (b) $2x - 5$ (c) $x - 10$ (d) $2x - 10$

8. Three times the square of the number x is .....
(a) $(3x)^2$ (b) $x^2 + 3$ (c) $3x^2$ (d) $\frac{x^2}{3}$

Essay problems:

1. A positive integer whose square is more than five times the number by 36

Find the number.

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2. An integer , if we add twice its square to the number 7 the result will be 135

Find the number.

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3. Find the real number whose double exceeds its multiplicative inverse by one.

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4. Find two real numbers whose product is 45 and one of them is 4 more than the other.

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5. The sum of the squares of two successive odd numbers is 130

Find the two numbers.

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6. The sum of three successive integers is equal to the square of their middle integer.

Find these numbers.

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7. Hatem is 4 years older than Hanan now , and the sum of squares of their ages now is 26

Find their ages now.

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8. A right-angled triangle, the lengths of the two sides of the right angle are $4x$ cm. and $x + 1$ cm. If the area of the triangle = 84 cm^2 , calculate the length of its hypotenuse.

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Homework .

9. What is the real number which exceeds its multiplicative inverse by $\frac{5}{6}$?

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10. Find the rational number whose four times its square equals 81

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11. What is the real number if it is added to its square , the result will be 12 ?

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12. Find the dimensions of a rectangle whose length is 4 cm. more than its width and whose area is 21 cm^2

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Lesson (10)
Integer powers in $\mathbb{R}$

Non-negative integer powers in $\mathbb{R}$

If $a \in \mathbb{R}$, $n \in \mathbb{Z}^+$, then $a^n = a \times a \times a \times a \times a \times \cdots \times a$ where a is repeated as a factor n times.
The symbol (a^n) is read as : a to the power n or the n^{th} power of the number a or the base a

Negative integer powers in $\mathbb{R}$

If a is a real number , $a \neq 0$ and n is a positive integer , then :

$$a^{-n} = \frac{1}{a^n} \quad \text{and} \quad a^n = \frac{1}{a^{-n}}$$

If $a \in \mathbb{R}^*$ (The set of non-zero real numbers) , then : $a^0 = 1$

$(-a)^n = a^n$ if n is an even number

$(-a)^n = -a^n$ if n is an odd number

Remarks

1 For every $a \in \mathbb{R}^*$, $n \in \mathbb{Z}^+$, then $a^n \times a^{-n} = a^n \times \frac{1}{a^n} = 1$ (the multiplicative neutral)

i.e. a^n and a^{-n} are the multiplicative inverse of each other.

2 For every $a \in \mathbb{R}^*$, $b \in \mathbb{R}^*$ and $n \in \mathbb{Z}^+$, then $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$

For example : $\left(\frac{2}{3}\right)^{-2} = \left(\frac{3}{2}\right)^2 = \frac{9}{4}$

Complete each of the following :

1. $(a^2 b^{\cdots\cdots})^4 = a^8 b^{12}$
2. If $(x-5)^{zero} = 1$, then : $x \in$ .....
3. If $a = 7^x$ and $b = 7^{-x}$, then : $a \times b =$ .....
4. If $x = \left(\sqrt{2}+3\right)^5$ and $y = \left(\sqrt{2}+3\right)^{-5}$, then : $x\,y =$ .....
5. $\left(\frac{5}{6}\right)^{-4} = \left(-\frac{\cdots\cdots}{\cdots\cdots}\right)^2$

6. If $\left(\frac{1}{2}\right)^{x} = 5$, then : $(8)^{-x} =$ .....

7. If $2^{x} = 7$, $2^{y} = 5$, then : $2^{x+y} =$ .....

8. If $5^{x} = 3$, $5^{-y} = 7$, then : $5^{x+y} =$ .....

Choose the correct answer :

1. $5^2 + 5^2 =$ .....

(a) 10^2 (b) 10^4 (c) 5^4 (d) 50

2. $3^5 \times 2^5 =$ .....

(a) 5^{10} (b) 6^{10} (c) 6^5 (d) 6^{25}

3. $(5\ a)^{zero} =$ ..... , $a \neq 0$

(a) 5 (b) a (c) 5 a (d) 1

4. $3\ x^{zero} =$ ..... , $x \neq 0$

(a) zero (b) 1 (c) 3 (d) $3\ x$

5. $3^{(2^3)} =$ .....

(a) 3^6 (b) 3^5 (c) 3^8 (d) 3^{32}

6. 🕮 $4^3 + 4^3 + 4^3 + 4^3 =$ .....

(a) 4^3 (b) 4^4 (c) 4^{12} (d) 4^{81}

7. The quarter of the number $4^{20} =$ .....

(a) 1^{20} (b) 4^{19} (c) 4^{16} (d) 4^5

8. 4 times the number $2^8 =$ .....

(a) 2^{32} (b) 8^8 (c) 2^{10} (d) 4^8

9. $\left(\sqrt{3}\right)^6 \times 3^4 =$ .....

(a) $\left(\sqrt{3}\right)^{24}$ (b) 3^{10} (c) 3^7 (d) $\left(\sqrt{3}\right)^{10}$

10. 🕮 The value of : $2^{20} + 2^{21} =$ .....

(a) 2×2^{40} (b) 2×2^{41} (c) 3×2^{20} (d) 3×2^{21}

11. 🕮 What of the following is closest to $11^2 + 9^2$?

(a) 22 + 18 (b) 211 + 29 (c) 120 + 20 (d) 120 + 80

12. 🕮 If $5^{x} = 4$, then 5^{x-1} = ...

(a) 1.25 (b) 0.8 (c) 0.125 (d) 0.08

13. 🕮 0.002×0.05 = ...

(a) 10^{-5} (b) 10^{-4} (c) 10^{4} (d) 10^{5}

14. $x^{m-1} \times$ ..... $= 1$, $x \neq 0$

(a) x^{m-1} (b) x^{-m-1} (c) x^{m+1} (d) x^{-m+1}

15. $5 \times 5 \times 5 \times 2 \times 2 \times 2 \times 2 \times 2 = 4 \times$ .....

(a) 5^3 (b) 2^3 (c) 10^3 (d) $5^3 + 2^3$

Homework

1. $(5^2)^3$ = ...

(a) 5^6 (b) 5^5 (c) 5^{32} (d) 5

2. $2^5 + 2^5 + 2^5 + 2^5$ = ...

(a) 2^4 (b) 2^6 (c) 2^7 (d) 2^{20}

3. 🕮 Sixth the number $2^{12} \times 3^{12}$ is .....

(a) 6^2 (b) 6^4 (c) 6^{11} (d) 6^{23}

4. Fifth the number $\left(\sqrt[3]{5}\right)^6$ is .....

(a) 5 (b) 5^5 (c) 5^6 (d) 5^{12}

5. 🕮 The value of : $2^5 + \left(\sqrt{2}\right)^{10}$ = ...

(a) 2^6 (b) 2^{10} (c) $\left(\sqrt{2}\right)^{15}$ (d) $\left(\sqrt{2}\right)^{20}$

6. If $6^{x} = 11$, then 6^{x+1} = ...

(a) 12 (b) 22 (c) 66 (d) 72

7. 🕮 If $x = \frac{\sqrt{9}}{\sqrt{3}}$, then x^{-1} = ...

(a) $\frac{\sqrt{3}}{3}$ (b) $\frac{\sqrt{3}}{\sqrt{2}}$ (c) $\sqrt{3}$ (d) 2

8. 🕮 $(\sqrt{3}+\sqrt{2})^{9}(\sqrt{3}-\sqrt{2})^{9}$ = ...

(a) 1 (b) $\sqrt{5}$ (c) $\sqrt{6}$ (d) 5

9. 🕮 The numerical value of the expression : $\frac{2^{2n+1}\times 5^{2n+1}}{10^{2n}}$ is .....

(a) $\frac{1}{10}$ (b) 7 (c) 10 (d) 100

10. $2^{2011} = 2^{2010} +$ .....

(a) 2 (b) 2010 (c) 2^{2010} (d) 2^{2011}

Find the value of each of the following in the simplest form:

1. 3^{-2}

2. $(\sqrt{5})^{4}$

3. $(-\sqrt{3})^{-2}$

4. $(0.01)^{-2}$

5. $(x^{2})^{-3}\times(x^{-3})^{-2}$

6. $\frac{(x^{2})^{-3}\times(x^{-1})^{2}}{x^{-3}\times x^{-4}}$

7. $(-\sqrt{5})^{9}\div(-\sqrt{5})^{5}$

8. $\left((\sqrt{2})^{3}\times(-\sqrt{2})^{2}\right)^{2}$

9. $(\sqrt{3})^{-4}\times(-\sqrt{2})^{4}$

10. $\left((-5)^{3}\right)^{2}\times(-\sqrt{5})^{-4}$

11. $\dfrac{(\sqrt{7})^{-4} \times (\sqrt{7})^{-3}}{(\sqrt{7})^{-9}}$

12. $\dfrac{(\sqrt{3})^{5} \times (\sqrt{3})^{4}}{(\sqrt{3})^{3} \times 27}$

13. $\dfrac{(10)^{2} \times (10)^{-7}}{(0.1)^{2} \times 0.001}$

14. $\left(\dfrac{3\sqrt{2}}{2\sqrt{3}}\right)^{4}$

15. $\dfrac{9^{x} \times 3^{x+2}}{(27)^{x}}$

16. $\dfrac{(36)^{n} \times 5^{2n}}{(30)^{2n}}$

17. $\dfrac{8^{n-1} \times 32^{-n}}{32 \times 4^{-n}}$

18. $\dfrac{6^{n} \times 4^{n+\frac{1}{2}}}{(24)^{n}}$

19. If $\dfrac{8^{x} \times 9^{x}}{18^{x}} = 64$, find the value of 4^{-x}

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20. If $a = \sqrt{3}$ and $b = \sqrt{2}$, find the value of :

1 $a^4 - b^4$

2 $\dfrac{a^4}{b^4}$

21. If $x = 2\sqrt{2}$ and $y = 3$, find the value of : $(x^2 - y^2)^3$

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Homework

1. $\left(\frac{1}{4}\right)^{-1}$

2. $\left(\frac{\sqrt{3}}{3}\right)^{-5}$

3. $x^3 \times x^{-2} \times x^{-1}$

4. $\left(\sqrt{2}\right)^2 \times \left(\sqrt{2}\right)^4$

5. $\left(\frac{-1}{\sqrt{2}}\right)^6$

6. $\frac{\left(\sqrt{3}\right)^7 \times \left(\sqrt{3}\right)^8}{\left(\sqrt{3}\right)^6}$

7. $\frac{\left(\sqrt{5}\right)^{10} \times \left(-\sqrt{5}\right)^5}{\left(\sqrt{5}\right)^{11}}$

8. $\frac{2^x \times 4^{x+1}}{8^x}$

9. $\frac{4^n \times 6^{2n}}{2^{4n} \times 3^{2n}}$

Lesson (11)
Solving exponential equations in R

If a is a real number , m and n are two integers

and $\boxed{a^m = a^n}$, then $\boxed{m = n}$ where : $a \neq 0$, $a \neq \pm 1$

For example :

If $3^n = 9$, then : $3^n = 3^2$

, ∵ the base = the base ∴ the power = the power ∴ $\boxed{n = 2}$

If a and b are two real numbers , m is an integer and $\boxed{a^m = b^m}$, then :

- $\boxed{a = b}$ if m is an odd number. ***For example :*** If $n^5 = 3^5$, then : $n = 3$
- $\boxed{a = \pm b}$ if m is an even number. ***For example :*** If $n^2 = 3^2$, then : $n = \pm 3$
- $\boxed{m = \text{zero}}$ if $a \neq \pm b$

For example : If $7^{n-2} = 5^{n-2}$, then : $n - 2 = 0$ ∴ $n = 2$

Complete each of the following :

1. If $5^{x(x-1)} = 1$, then the value of x = ...
2. If $3^n \times 3^5 = 1$, then n = ...
3. If $3^x + 3^x + 3^x = 1$, then x = ...

   .....

   .....
4. If $\{3 , a^{x-2}\} = \{1 , 3\}$, then the value of x = ...
5. If $(2^x , 125) = (16 , y^3)$, then x = ... and y = ...

Homework

1. If $2^y \times 5^y = 100$, then y = ...
2. If $\left(\frac{3}{5}\right)^{x-7} = 1$, then x = ...

Choose the correct answer :

1. If $3^{x+1} = 5^{x+1}$, then $x =$ .....

(a) 4 (b) 3 (c) – 1 (d) 1

2. If $3^{2+x} = 5^{x+2}$, then $7^{x+2} =$ .....

(a) 7 (b) – 7 (c) – 14 (d) 1

3. If $\left(\frac{2}{3}\right)^{9} = \left(\frac{3}{2}\right)^{x}$, then $x =$ .....

(a) – 9 (b) 9 (c) 32 (d) 23

4. If $5^{|x-3|} = 25$, then $x =$ .....

(a) 5 (b) 2 (c) 1 (d) 5 or 1

5. If $2^{x-1} \times 3^{1-x} = \frac{9}{4}$, then $x =$ .....

(a) – 3 (b) – 1 (c) 1 (d) 3

Homework

1. If $2^{x} = \frac{1}{8}$, then $x^{2} =$ .....

(a) $\frac{1}{4}$ (b) 9 (c) – 9 (d) $-\frac{1}{9}$

2. If $2^{x-2} = 2^{1-2x}$, then $x =$ .....

(a) 2 (b) $\frac{1}{2}$ (c) 1 (d) zero

3. If $3^{x} = 9$, then $2^{x} - 1 =$ .....

(a) 7 (b) 3 (c) 8 (d) 5

4. If $3^{x} = 7$, $7^{y} = 9$, then $x\,y =$ .....

(a) 5 (b) 2 (c) 7 (d) 9

Essay problems:

Find the value of n in each of the following when $n \in \mathbb{Z}$:

1. $3^{n-2} = 81$

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2. $\left(\sqrt{3}\right)^{n-1} = 9$

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3. $\left(\frac{3}{5}\right)^{n+2} = \frac{125}{27}$

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4. $\left(\frac{2}{3}\right)^{n-4} = 2\frac{1}{4}$

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5. $\left(\frac{2}{3}\right)^{n+5} = \left(3\frac{3}{8}\right)^{-2}$

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6. $\frac{2^n \times 9^{n+1}}{(18)^n} = 3^n$

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7. $\frac{(12)^{n-1}}{2^{n-1} \times 3^{n-1}} = 1$

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8. $\frac{(14)^{2n} \times 4^{n+1}}{4 \times 7^n \times 16^n} = 49$

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Homework

1. $3^{n-2} = \frac{1}{9}$

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2. $\left(\frac{2}{5}\right)^{2n-1} = \frac{8}{125}$

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3. $\frac{3^n \times 8^n}{(12)^{n+1}} = \frac{1}{3}$

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Find the S.S. of each of the following equations in $\mathbb{R}$:

1. $3^{x-3} = \left(\sqrt{3}\right)^{x+5}$

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2. $25 \times 3^{x-1} = 9 \times 5^{x-1}$

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3. $5^{x^2-5x} = 0.0016$

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4. $\frac{1}{(x+9)^4} = 0.0001$

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5. $9^{x^2-1} = \frac{1}{(27)^x}$

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6. If $\left(\sqrt{\frac{3}{2}}\right)^x = \frac{4}{9}$, calculate the value of : $\left(\frac{3}{2}\right)^{x+1}$

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Homework

1. $(32)^{x-3} = 8^{2x+1}$

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2. If $\frac{49^n \times 25^{2n} \times 3^{4n}}{7^{-n} \times 15^{4n}} = 343$, then calculate the value of : 6^{2n}

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3. If $3^x = 27$, $4^{x+y} = 1$, calculate the value of each of : x and y

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Lesson (12)
Operations on integer powers

1. Do the operations inside the parenthesis (the interior, then the exterior).
2. Calculate the powers of the numbers (indices).
3. Do the multiplication and the division in order from left to right.
4. Do the addition and the subtraction in order from left to right.

Complete each of the following :

1. The simplest form of the expression : $2^{-3} \times 2^{-2} \div 4^{-3} =$ .....

2. The simplest form of the expression : $4^3 \times 3^{-2} \times \left(\sqrt[3]{-8}\right)^{-5} =$ .....

Homework

1. The simplest form of the expression : $2^{-3} \times 3^{-2} \div 6^{-4} =$ .....

2. The simplest form of the expression : $\left(3^{-2}\right)^3 \div 9^{-3} \times (-2)^{-1} =$ .....

Choose the correct answer :

1. The expression : $\dfrac{3^x \times 3^x \times 3^x}{3^x + 3^x + 3^x}$ equals .....

(a) 3^{2x-1} (b) 3^{1-2x} (c) 3^{x^3-3x} (d) 3^{3x-x^3}

2. $(5^{x+2} - 5^{x+1}) \div 5^x =$ .....

(a) 5 (b) 10 (c) 15 (d) 20

Homework

1. The value of the expression : $3^5 + \left(\sqrt{3}\right)^{10} - 2\,(3)^5 =$ .....

(a) zero (b) 3^5 (c) $\left(\sqrt{3}\right)^5$ (d) $2\,(3)^5$

2. The simplest form of the expression : $\sqrt{4 \times \sqrt{16} \div \sqrt[3]{8} - 2^2} =$ .....

(a) 2 (b) 4 (c) 8 (d) 16

3. If $x = \sqrt{3}$, $y = \sqrt{5}$, then : $\dfrac{x^8 - y^8}{x^4 + y^4} =$ .....

(a) 4 (b) – 4 (c) 16 (d) – 16

Find the result in the simplest form:

1. $(\sqrt{5})^5 \div 5\sqrt{5} + 2\sqrt{3} \times \sqrt{3}$

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2. $(\sqrt{3})^{-3} \times 3\sqrt{3} + (\sqrt{3})^{-4} \div (\sqrt{3})^{-10}$

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3. $$\frac{(\sqrt{3})^7 \times (\sqrt{3})^{-5} - (\sqrt{3})^2}{(\sqrt{3})^7 \times (\sqrt{3})^{-5} + (\sqrt{3})^2}$$

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4. 🕮 **If $a = \sqrt{2}$, $b = \sqrt{3}$, find the numerical value of :**

1 $\frac{b^4 - a^4}{b^2 + a^2}$ « 1 »

2 $\frac{a^3 + b^3}{a + b}$ « $5 - \sqrt{6}$ »

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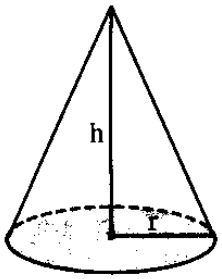
5. 🕮 If the volume of the right circular cone is given by the relation : $v = \frac{1}{3} \pi r^2 h$

Find the height of the cone h if the volume is : 7.7×10^2 cm^3 and its diameter length is 14 cm. $\left(\pi = \frac{22}{7}\right)$



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Homework

1. $\left(2\sqrt{3}\right)^3 \times \sqrt{3} - \left(\sqrt{2}\right)^7 \div 4\sqrt{2}$

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2. $\left(2\sqrt{5}\right)^4 - \left(\sqrt{5}\right)^{-3} \times \left(5\sqrt{5}\right)^2 \div 5\sqrt{5}$

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Lesson (16)
Areas of some geometric figures

The area of the rhombus = L × h where L is the side length and h is the height.

The area of the rhombus = $\frac{1}{2}$ of the product of the lengths of its two diagonals.

If the two legs of the trapezium are equal in length, then it is called an isosceles trapezium.
The following are the properties of the isosceles trapezium :

The two base angles of the isosceles trapezium are equal in measure.

The two diagonals of the isosceles trapezium are equal in length.

The isosceles trapezium has only one axis of symmetry which is the perpendicular bisector of its bases.

The area of the trapezium = half of the sum of lengths of the two parallel bases × height

The area of the trapezium = the length of the middle base × height

Complete each of the following :

1. The area of rhombus whose perimeter is 20 cm. and height 4 cm. = ...
2. The length of the diagonal of a square of area 50 cm^2 equals ..... cm.
3. The length of side of a square whose area equals the area of a rectangle with dimensions 9 cm. , 16 cm. = ...
4. The length of the middle base of a trapezium whose area = 30 cm^2 and height 5 cm. equals .....

Homework

1. The area of the rhombus = the side length × ..... = $\frac{1}{2}$ of the product of .....
2. The area of the square = the square of the length of ..... = $\frac{1}{2}$ .....
3. The length of the middle base of the trapezium equals .....
4. The area of the trapezium = half of the sum of lengths of the two parallel bases × .....
   = the length of ..... × its height
5. The base angles of the isosceles trapezium are .....

6. The diagonals of an isosceles trapezium are .....

Choose the correct answer :

1. If the area of a square is 50 cm^2, then the length of its diagonal = ...

(a) 25 cm. (b) 5 cm. (c) 10 cm. (d) 20 cm.

2. If the perimeter of a rhombus is 24 cm. and its area = 30 $cm.^2$ then its height = ...

(a) 4 cm. (b) 5 cm. (c) 6 cm. (d) 12 cm.

3. If the product of the lengths of the diagonals of a rhombus = 96 cm^2 and its height is 6 cm., then its side length = ...

(a) 12 cm. (b) 8 cm. (c) 6 cm. (d) 4 cm.

4. If the area of a trapezium is 32 cm^2 and its height is 4 cm., then the length of its middle base = ...

(a) 4 cm. (b) 8 cm. (c) 14 cm. (d) 16 cm.

5. The trapezium in which the length of one of its parallel bases is 15 cm., and its area is 108 cm^2 and its height is 8 cm., then the length of the other base is .....

(a) 15 cm. (b) 4 cm. (c) 12 cm. (d) 27 cm.

6. The trapezium whose middle base length is x cm. and its height = $\frac{1}{2}$ the length of the middle base, its area = ... cm^2

(a) x^2 (b) $\frac{x^2}{2}$ (c) $\frac{x^2}{4}$ (d) $\frac{x^2}{8}$

Homework

1. The area of rhombus is 20 cm^2, the length of one of its diagonals is 5 cm., then the length of the other diagonal = ...

(a) 8 cm. (b) 4 cm. (c) 10 cm. (d) 15 cm.

2. The area of the square whose side length is 6 cm. ..... the area of the square whose diagonal length is 8 cm.

(a) > (b) < (c) = (d) ≡

3. The trapezium in which the lengths of its two parallel bases are 15 cm. and 11 cm. Its middle base is with length .....

(a) 26 cm. (b) 15 cm. (c) 13 cm. (d) 11 cm.

4. If the area of the trapezium is 450 cm^2, and the lengths of its two parallel bases are 24 cm. and 12 cm. , then its height = ...

(a) 12.5 cm. (b) 25 cm. (c) 36 cm. (d) 52 cm.

Find the area of the following figures:

1. A rhombus of side length 6 cm. and its height = 5 cm. « 30 cm^2 »

.....

2. A rhombus whose diagonal lengths are 24 cm. and 10 cm. « 120 cm^2 »

.....

3. A square whose diagonal length = 10 cm. « 50 cm^2 »

.....

4. A trapezium whose bases lengths are 8 cm. and 10 cm. and its height = 5 cm. « 45 cm^2 »

.....

5. A trapezium whose middle base length is 7 cm. and its height = 6 cm. « 42 cm^2 »

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Homework

1. A rhombus whose side length 12 cm. and its height = 8 cm. « 96 cm^2 »

.....

2. A rhombus whose diagonals lengths are 8 cm. and 10 cm. « 40 cm^2 »

.....

3. A square whose diagonal length = 8 cm. « 32 cm^2 »

.....

4. A trapezium whose bases lengths are 6 cm. and 8 cm. and its height = 12 cm. « 84 cm^2 »

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5. A trapezium whose middle base length is 12 cm. and its height = 8 cm. « 96 cm^2 »

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Essay problems:

1. A square whose area equals the area of the rectangle whose dimensions are 2 cm. and 9 cm. Find the length of its diagonal. « 6 cm. »

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2. 🕮 Two pieces of land have equal areas , one of them has the shape of a rhombus whose diagonals are 18 m. and 24 m. , and the other one has the shape of a trapezium whose height is 12 m. Find the length of its middle base. « 18 m. »

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3. 🕮 The area of a trapezium is 180 cm^2 and its height is 12 cm. Find the lengths of its parallel bases if the ratio between their lengths is 3 : 2 « 18 cm. , 12 cm. »

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Homework

1. Two land pieces are equal in area , the first is in the shape of a square and the second is in the shape of a rhombus whose diagonals lengths are 8 metres and 16 metres. Find the perimeter of the square-shaped piece. « 32 cm. »

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2. 🕮 Find the area of the rhombus whose perimeter is 52 cm. and the length of one of its diagonals is 10 cm. « 120 cm^2 »

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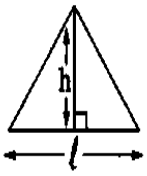
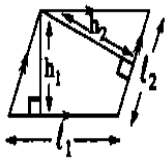
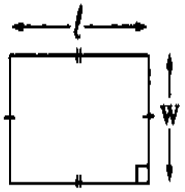
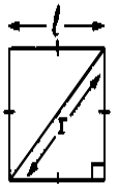
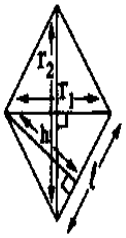
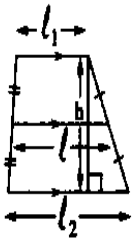
The figure		The perimeter	The area
The triangle		The sum of the lengths of its three sides	$\frac{1}{2}$ of the base length × height $= \frac{1}{2} l \times h$
The parallelogram		The sum of lengths of two adjacent sides × 2 $= 2(l_1 + l_2)$	The base length × height $= l_1 \times h_1 = l_2 \times h_2$
The rectangle		2 (Length + Width) $= 2(l + W)$	Length × Width $= l \times W$
The square		Side length × 4 $= 4l$	Square of side length $= l^2$ or $\frac{1}{2}$ of the square of its diagonal length $= \frac{1}{2} r^2$
The rhombus		Side length × 4 $= 4l$	Side length × height $= l \times h$ or $\frac{1}{2}$ the product of the lengths of the two diagonals $= \frac{1}{2} r_1 \times r_2$
The trapezium		The sum of lengths of its sides	$\frac{1}{2}$ the sum of lengths of the two parallel bases × height $= \frac{1}{2}(l_1 + l_2) \times h$ or the length of the middle base × height $= l \times h$

Lesson (17)
Similarity

It is said that the two polygons P_1 and P_2 (of the same number of sides) are similar if the following two conditions are verified together :

1. Their corresponding angles are equal in measure.
2. The corresponding side lengths are proportional.

In this case , we write the polygon $P_1 \sim$ the polygon P_2

That means the polygon P_1 is similar to the polygon P_2

Remark (1)

In the two similar polygons P_1 and P_2 , the constant ratio among the lengths of the corresponding sides of P_1 and P_2 is called the ratio of enlargement or the drawing scale.

If the constant ratio is :

- Greater than 1 , then the polygon P_1 is an enlargement to the polgyon P_2
- Less than 1 , then the polygon P_1 is a minimizing of the polgyon P_2
- Equal to 1 , then the polygon P_1 is congruent to the polgyon P_2

Remark (2)

In order that two polygons are similar , the two conditions should be verified together and verifying one of them only is not enough to be similar.

Remark (3)

The congruent polygons are similar but it is not necessary that the similar polygons are congruent.

Remark (4)

All regular polygons of the same number of sides are similar.

Remark (5)

If each of two polygons is similar to a third polygon , then they are similar.

Remark (6)

The order of corresponding vertices should be kept in giving names of similar polygons that to help us finding the proportional sides lengths and the equal angles in measures.

i.e.

The ratio between the perimeters of two similar polygons = the ratio between the lengths of two corresponding sides.

A geometric fact :

The two triangles are similar if one of the two following conditions is verified :

1. The measures of their corresponding angles are equal.
2. The lengths of their corresponding sides are proportional.

Remarks

1. **The two right-angled triangles are similar if the measure of an acute angle in one of them is equal to the measure of an acute angle in the other.**
2. **The two equilateral triangles are similar.**
3. **The two isosceles triangles are similar if the measure of an angle in one of them equals the measure of the corresponding angle in the other.**

Complete each of the following :

1. If the measures of the corresponding angles in the two triangles are equal , then the two triangles are .....
2. If we have two polygons , their corresponding angles are ..... and their corresponding sides lengths are ..... , then the two polygons are similar.
3. If the ratio between the lengths of two corresponding sides in two similar triangles is equal to 1 , then the two triangles are .....
4. If two polygons are similar and the ratio between the lengths of two corresponding sides is 3 : 4 , then the ratio between their perimeters is .....

Homework

1. If two polygons are similar , then the corresponding ..... are equal in measure.
2. If two polygons are similar , then the corresponding ..... are proportional.
3. If each of two polygons is similar to a third , then they are .....
4. The two triangles are similar if the corresponding ..... are proportional.

Choose the correct answer :

1. If the ratio between the lengths of two corresponding sides of two squares is 1 and the perimeter of one of them is 20 cm. , then the area of the other square = ...

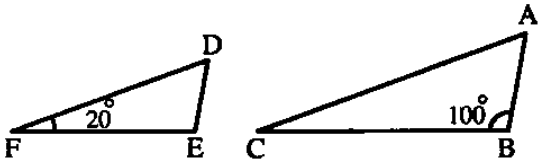
(a) 20 cm^2 (b) 25 cm^2 (c) 16 cm^2 (d) 25 cm.

2. **In the opposite figure :**

If $\Delta ABC \sim \Delta DEF$, then m ($\angle A$) = ...

(a) 20° (b) 60°

(c) 80° (d) 100°



3. **In the opposite figure :**

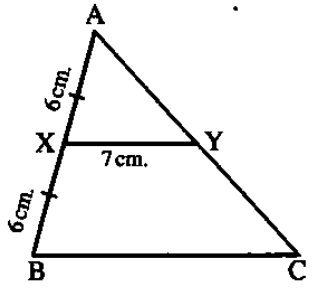
If $\Delta ABC \sim \Delta AXY$,

AX = XB = 6 cm.

XY = 7 cm. , then BC = ...

(a) 6 cm. (b) 7 cm.

(c) 12 cm. (d) 14 cm.

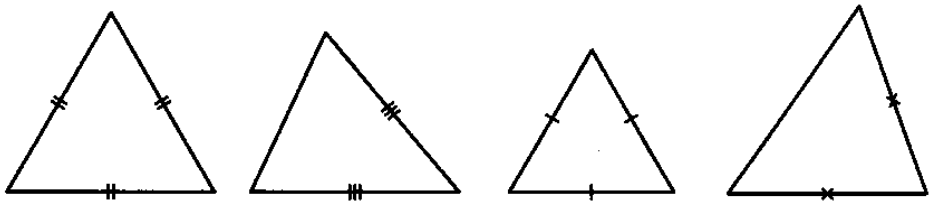


Homework

1. If $\Delta ABC \sim \Delta DEF$ and $AB = \frac{1}{5} DE$, then perimeter of ΔABC = ... perimeter of ΔDEF

(a) 5 (b) 1 (c) $\frac{1}{5}$ (d) $\frac{2}{5}$

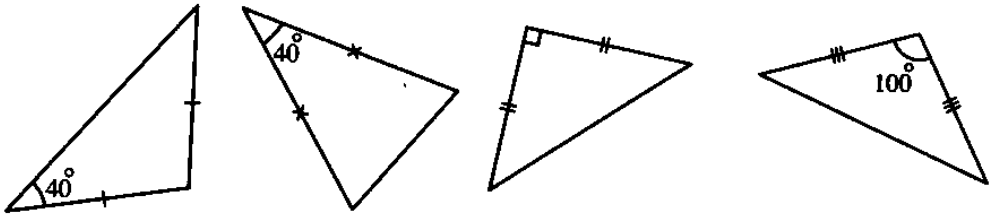
2. In the following figures , there are two similar triangles , they are .....



(1) (2) (3) (4)

(a) 1 , 2 (b) 1 , 3 (c) 1 , 4 (d) 2 , 4

3. In the following figures , there are two similar triangles , they are .....



(1) (2) (3) (4)

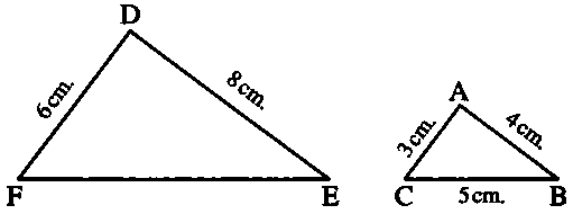
(a) 1 , 2 (b) 1 , 3 (c) 2 , 4 (d) 1 , 4

4. **In the opposite figure :**

If Δ ABC ~ Δ DEF , then EF = ...

(a) 5 cm. (b) 6 cm.

(c) 8 cm. (d) 10 cm.

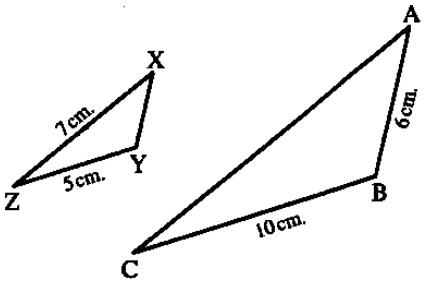


Essay problems:

1. **In the opposite figure :**

Δ ABC ~ Δ XYZ

Find : AC and XY « 14 cm. , 3 cm. »



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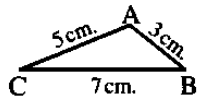
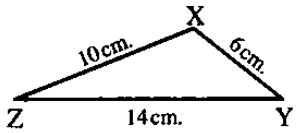
In the opposite figure :



1 **Prove that :** Δ ABC and Δ XYZ are similar.

2 If : m (∠ B) + m (∠ C) = 60° ,

find : m (∠ X) « 120°»



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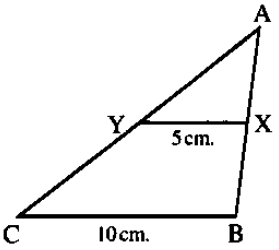
3.

In the opposite figure :

If Δ AXY ~ Δ ABC

XY = 5 cm. and BC = 10 cm. ,

Prove that : **1** $\overline{XY}$ // $\overline{BC}$ **2** Y is the midpoint of $\overline{AC}$



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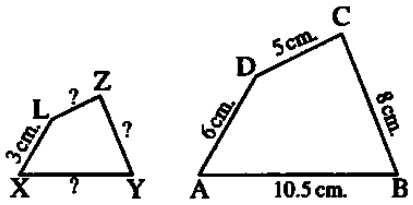
4. **In the opposite figure :**

The polygon ABCD ~ the polygon XYZL

If AB = 10.5 cm. , BC = 8 cm. , CD = 5 cm. ,

DA = 6 cm. and LX = 3 cm.



Find the length of each of : $\overline{XY}$, $\overline{YZ}$ and $\overline{ZL}$

« 5.25 cm. , 4 cm. , 2.5 cm. »

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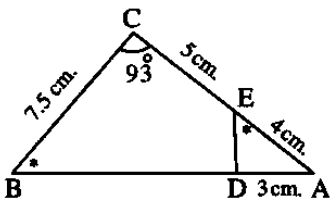
5. **In the opposite figure :**

ΔABC , $D \in \overline{AB}$, $E \in \overline{AC}$

, AE = 4 cm. , EC = 5 cm. , BC = 7.5 cm.

, AD = 3 cm. , m (∠ AED) = m (∠ B) and m (∠ C) = 93°



1 **Prove that :** $\Delta AED \sim \Delta ABC$

2 **Find the length of each of :** $\overline{BD}$ and m (∠ ADE)

« 9 cm. , 93° »

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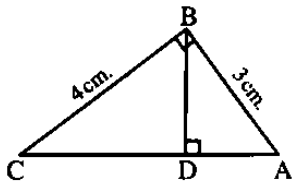
6. 📖 **In the opposite figure :**

ABC is a right-angled triangle at B in which :

AB = 3 cm. , BC = 4 cm. and $\overline{BD} \perp \overline{AC}$



1 Prove that : $\Delta BAC \sim \Delta DAB$

2 Find the length of each of : $\overline{AD}$ and $\overline{DC}$ « 1.8 cm. , 3.2 cm. »

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7. 📖 Two similar triangles, one of them has a perimeter of 74 cm. and the sides lengths of the other are 4.5 cm. , 6 cm. and 8 cm.

Find the length of the longest side in the first triangle. « 32 cm. »

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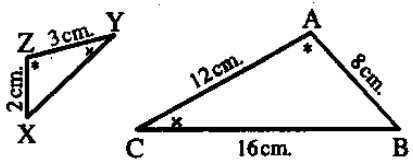
Homework

1. **Using the shown data in the figure , then prove that :**

Δ XYZ and Δ BCA

are similar , then find

the perimeter of Δ XYZ « 9 cm. »



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2. **In the opposite figure :**

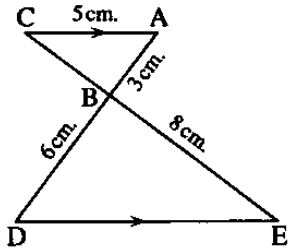
$\overline{AC} // \overline{ED}$, $\overline{AD} \cap \overline{CE} = \{B\}$

, AC = 5 cm. , BE = 8 cm. , AB = 3 cm. and BD = 6 cm.

1 **Prove that :** $\Delta ABC \sim \Delta DBE$

2 **Find the length of each of :** $\overline{BC}$ and $\overline{ED}$

3 **Find :** the ratio of enlargement.



« 4 cm. , 10 cm. , 2 »

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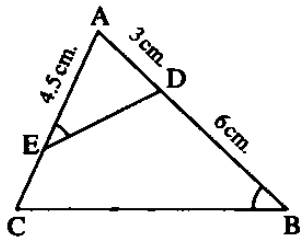
3. 📖 **In the opposite figure :**



$m(\angle AED) = m(\angle B)$, $AD = 3$ cm.

$AE = 4.5$ cm. and $BD = 6$ cm.

1 **Prove that :** $\Delta ADE \sim \Delta ACB$

2 **Find the length of :** $\overline{EC}$ « 1.5 cm. »

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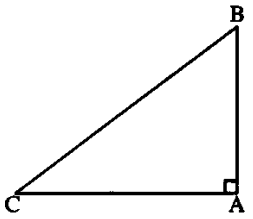
Lesson (18)
The converse of Pythagoras' theorem

We studied Pythagoras' theorem last year.

In the following , we will remind you of what you have studied.

If ABC is a right-angled triangle at A, then $(BC)^2 = (AB)^2 + (AC)^2$

Now we shall study the converse of Pythagoras' theorem.

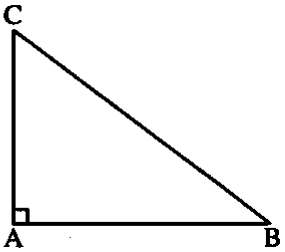


In a triangle , if the sum of the areas of two squares on two sides is equal to the area of the square on the third side, then the angle opposite to this side is a right angle.

In Δ ABC , if :

$(AB)^2 + (AC)^2 = (BC)^2$,

then : m (∠ A) = 90°



We can state this theorem as follows :

In a triangle, if the square of the length of a side is equal to the sum of the squares of the lengths of the other two sides, then the angle opposite to this side is a right angle.

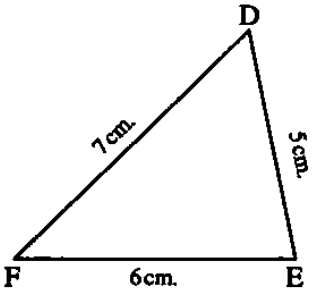
Corollary

In Δ ABC , if $\overline{AC}$ is the longest side and if $(AC)^2 \neq (AB)^2 + (BC)^2$, then m (∠ B) ≠ 90° and the triangle is not right-angled.

Complete each of the following :

Complete and show which of the following triangles is a right-angled triangle :

1

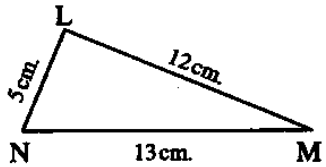


$(DF)^2$ = ...

$(DE)^2 + (EF)^2$ = ...

∴ The triangle is .....

2

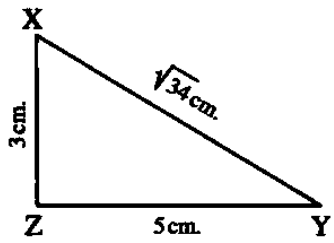


$(MN)^2$ = ...

$(ML)^2 + (NL)^2$ = ...

∴ The triangle is .....

3

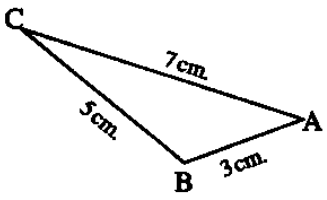


$(XY)^2 = (\sqrt{34})^2 =$ .....

$(YZ)^2 + (ZX)^2 =$ .....

∴ The triangle is .....

4



$(AC)^2 =$ .....

$(AB)^2 + (BC)^2 =$ .....

∴ The triangle is .....

Homework

In each of the following figures

Prove that : m (∠ B) = 90°

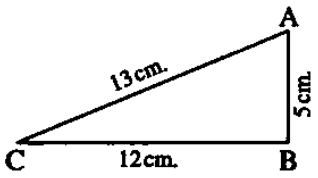


Fig. (1)

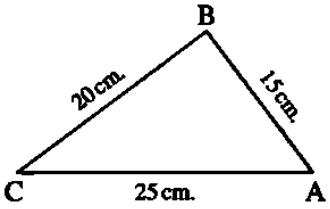


Fig. (2)

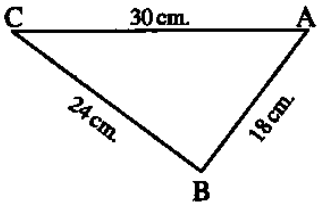


Fig. (3)

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Essay problems:

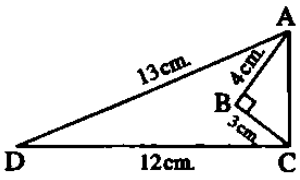
1.

In the opposite figure :

m (∠ B) = 90° , AB = 4 cm. , BC = 3 cm.

AD = 13 cm. and DC = 12 cm.

Find : The area of the figure ABCD



« 24 cm^2 »

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2.

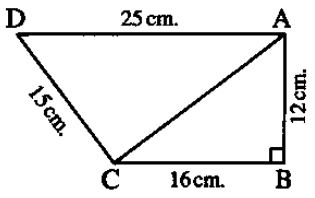
In the opposite figure :

ABCD is a quadrilateral in which : m (∠ B) = 90° ,

AB = 12 cm. , BC = 16 cm. , CD = 15 cm. and DA = 25 cm.

1 **Find :** The length of $\overline{AC}$

2 **Prove that :** m (∠ ACD) = 90°



« 20 cm. »

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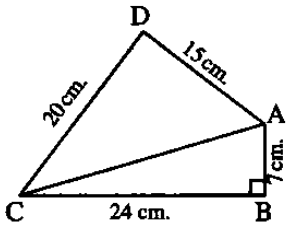
Homework

1 **In the opposite figure :**

ABCD is a quadrilateral in which : m (∠ ABC) = 90° ,

AB = 7 cm. , BC = 24 cm. , CD = 20 cm. and DA = 15 cm.

Prove that : m (∠ ADC) = 90°



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2. ABC is a triangle in which : AB = 4.5 cm. , BC = 7.5 cm. , AC = 6 cm.

Prove that : Δ ABC is right-angled.

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3.

In the opposite figure :

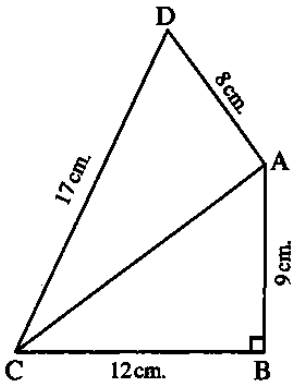
ABCD is a quadrilateral in which : m (∠ B) = 90° ,

AB = 9 cm. , BC = 12 cm. ,

CD = 17 cm. and DA = 8 cm.

Prove that : m (∠ DAC) = 90° ,

then find : The area of the figure ABCD « 114 cm^2 »



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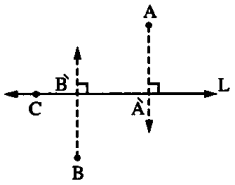
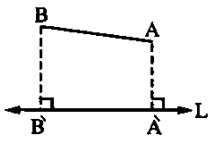
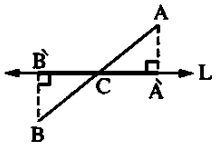
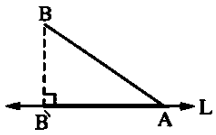
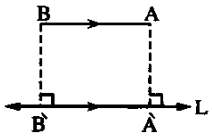
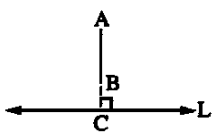
Lesson (19)
Projections

The projection of a point on a straight line:

1 The projection of a point on a straight line is the point of intersection of the perpendicular segment from this point and the straight line. 2 If the point lies on the straight line, its projection on it is the same point.	

The projection of a line segment on a straight line:

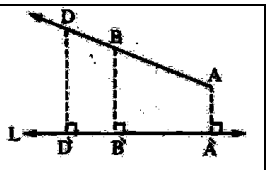
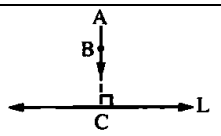
The projection of a line segment on a given straight line is the line segment whose two endpoints are the projections of the two endpoints of the main line segment on this straight line.

				
$\grave{A}\grave{B} < AB$	$\grave{A}\grave{B} < AB$	$A\grave{B} < AB$	$\grave{A}\grave{B} = AB$	$\grave{A}\grave{B}$ = zero

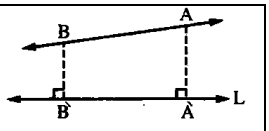
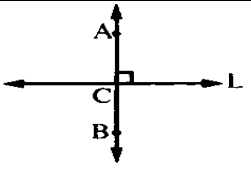
From the table, we notice that :

The length of the projection of a line segment on a given straight line $\leq$ the length of the line segment.

The projection of a ray on a straight line:

The projection of a ray on a straight line not perpendicular to it is a ray $\subset$ this straight line.	
The projection of a ray on a straight line perpendicular to it is a point belonging to the straight line.	

The projection of a straight line on another straight line:

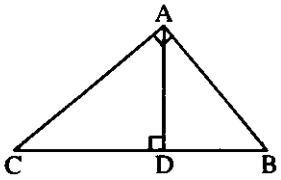
The projection of a straight line on a straight line not perpendicular to it is a straight line.	
The projection of a straight line on a straight line perpendicular to it is the point of intersection of the two straight lines.	

Complete each of the following :

1.

In the opposite figure :

ΔABC is right-angled at A and $\overline{AD} \perp \overline{BC}$



Complete the following :

1. The projection of $\overline{AB}$ on $\overleftrightarrow{BC}$ is .....
2. The projection of $\overline{AC}$ on $\overleftrightarrow{BC}$ is .....
3. The projection of $\overline{BC}$ on $\overleftrightarrow{AC}$ is .....
4. The projection of $\overline{BC}$ on $\overleftrightarrow{AB}$ is .....
5. The projection of $\overline{AC}$ on $\overleftrightarrow{AD}$ is .....
6. The projection of $\overline{AD}$ on $\overleftrightarrow{BC}$ is .....
7. The projection of $\overline{AB}$ on $\overleftrightarrow{AD}$ is .....

2. If $X \in \overleftrightarrow{AB}$, then the projection of X on $\overleftrightarrow{AB}$ is .....

3. If $\overline{AB} \perp \overline{BC}$, then the projection of $\overline{AB}$ on $\overleftrightarrow{BC}$ is .....

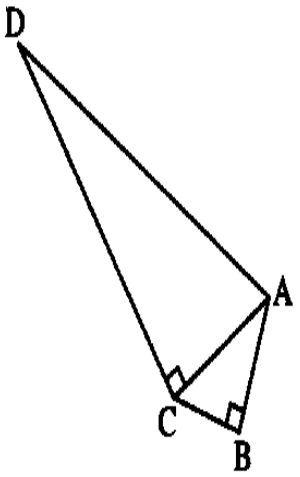
4. In ΔABC , if $m(\angle B) = 90°$, then the projection of C on $\overleftrightarrow{AB}$ is .....

5. ABC is a right-angled triangle at A , then the projection of $\overline{BA}$ on $\overleftrightarrow{AC}$ is .....

Homework

1. 🕮 **In the opposite figure :**

$m(\angle B) = m(\angle ACD) = 90°$



Complete :

1. The projection of $\overline{AD}$ on $\overleftrightarrow{CD}$ is .....
2. The projection of $\overline{AC}$ on $\overleftrightarrow{CD}$ is .....
3. The projection of $\overline{AC}$ on $\overleftrightarrow{AB}$ is .....

Choose the correct answer :

1. The projection of a ray on a straight line not perpendicular to it is .....

(a) a point. (b) a line segment. (c) a ray. (d) a straight line.

2. The length of the projection of a line segment on a given straight line ..... the length of the line segment itself.

(a) ≤ (b) > (c) ≥ (d) =

3. The length of the projection of a line segment on a straight line parallel to it ..... the length of the main line segment.

(a) < (b) > (c) = (d) ≠

4. The length of the projection of a line segment on a straight line perpendicular to it is .....

(a) greater than the length of the main line segment.

(b) equal to the length of the main line segment.

(c) greater than or equal to the length of the main line segment.

(d) equal to zero.

Homework .

1. The projection of a point on a given straight line is .....

(a) a point. (b) a line segment. (c) a ray. (d) a straight line.

2. The projection of a line segment on a straight line not perpendicular to it is .....

(a) a ray. (b) a point. (c) a line segment. (d) a straight line.

3. The projection of a line segment on a straight line perpendicular to it is .....

(a) a point. (b) a line segment. (c) a ray. (d) a straight line.

Essay problems:

1. 📖 **In the opposite figure :**

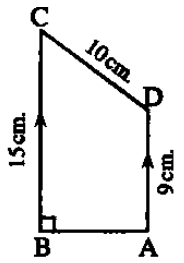
ABCD is a trapezium in which $\overline{AD}$ // $\overline{BC}$ and m (∠ ABC) = 90°

If AD = 9 cm. , DC = 10 cm. and CB = 15 cm.

Find :

1 The length of the projection of $\overline{DC}$ on $\overleftrightarrow{BC}$

2 The length of the projection of $\overline{DC}$ on $\overleftrightarrow{AB}$



« 6 cm., 8 cm. »

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Homework

1. 📖 **In the opposite figure :**

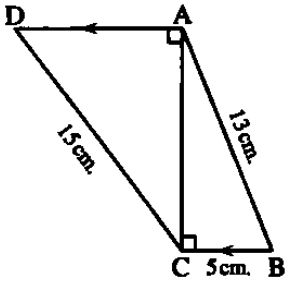
$\overline{AD}$ // $\overline{BC}$, AB = 13 cm. , BC = 5 cm. ,

CD = 15 cm. and m (∠ ACB) = m (∠ DAC) = 90°

Find :

1 The length of the projection of $\overline{AB}$ on $\overleftrightarrow{AC}$

2 The length of the projection of $\overline{CD}$ on $\overleftrightarrow{AD}$



« 12 cm., 9 cm. »

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Lesson (20)
Euclidean Theorem

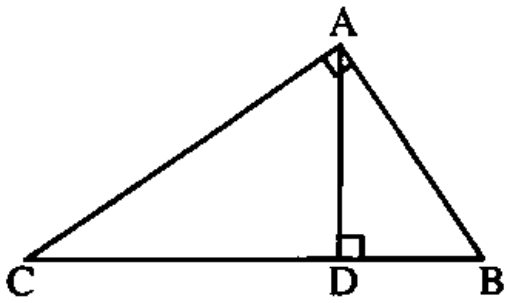
In the right-angled triangle , the area of the square on a side of the right angle is equal to the area of the rectangle whose dimensions are the length of the projection of this side on the hypotenuse and the length of the hypotenuse.

$(AB)^2 = DB \times BC$

$(AC)^2 = DC \times BC$

$(DA)^2 = DB \times DC$

$DA = \frac{BA \times AC}{BC}$

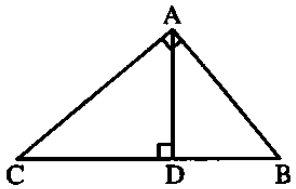


Complete each of the following :

1.

In the opposite figure :

Δ ABC is right-angled at A , $\overline{AD} \perp \overline{BC}$



Complete each of the following :

1 $(AC)^2$ = ... + ...

2 $(AC)^2$ = ... – .....

3 $(AC)^2$ = ... × .....

4 $(AD)^2$ = ... × .....

5 AC × AB = ... × .....

6 Δ ABC ~ Δ ..... ~ Δ .....

Homework

1. 🕮 **In the opposite figure :**

ABC is a triangle in which m ($\angle$ ABC) = 90° , AB = 4 cm. ,

AC = 5 cm. and $\overline{BD} \perp \overline{AC}$

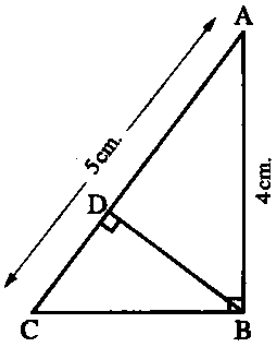
Complete :

1 BC = ... cm.

2 AD = ... cm.

3 BD = ... cm.

4 The area of Δ DBC = ... cm^2



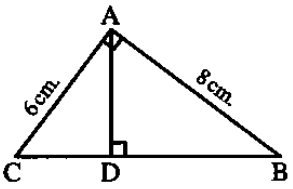
Essay problems:

1.

In the opposite figure :

ABC is a triangle in which m (∠ BAC) = 90° , $\overline{AD} \perp \overline{BC}$, AB = 8 cm. and AC = 6 cm.

Find : BD , CD and AD



« 6.4 cm. , 3.6 cm. , 4.8 cm. »

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2.

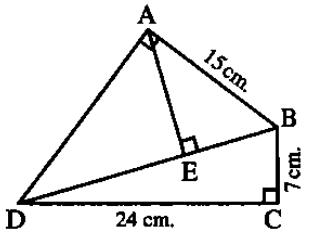
In the opposite figure :

ABCD is a quadrilateral where

m (∠ BCD) = m (∠ BAD) = 90° ,

$\overline{AE} \perp \overline{BD}$, BC = 7 cm. , CD = 24 cm.

and AB = 15 cm.



Find : 1 The length of each of $\overline{BD}$ and $\overline{AD}$

2 The length of the projection of $\overline{AB}$ on $\overleftrightarrow{BD}$

3 The length of the projection of $\overline{AD}$ on $\overleftrightarrow{AE}$

« 25 cm. , 20 cm. , 9 cm. , 12 cm. »

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3. **In the opposite figure :**

Δ ABC is right-angled at B

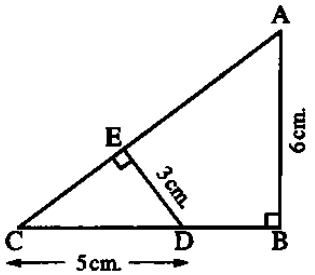
, $\overline{DE} \perp \overline{AC}$, AB = 6 cm.

, ED = 3 cm. and CD = 5 cm.

Prove that : Δ CED ~ Δ CBA

and find : The length of $\overline{AC}$

and the length of the projection of $\overline{AB}$ on $\overleftrightarrow{AC}$



« 10 cm. , 3.6 cm. »

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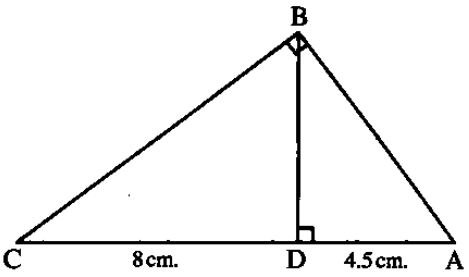
Homework

1. **In the opposite figure :**

Δ ABC is right-angled at B and $\overline{BD} \perp \overline{AC}$

If AD = 4.5 cm. and DC = 8 cm. ,

find : The length of each of $\overline{AB}$, $\overline{BC}$ and $\overline{BD}$



« 7.5 cm. , 10 cm. , 6 cm. »

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كيفية طباعة صفحات معينة من ملف معين

مثلا ازاى نطبع الصفحات من صفحة 4 الى صفحة 9